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First progress report on structural behavior of battledeck floor systems, 1935

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STRUCTURAL BEHAVIOR
OF BATTLEDECK FLOOR SYSTEMS

by Inge Lyse* and Ingvald E. Madsen**

1. SYNOPSIS

This report is a progress report on the first year's investigation of Battledeck Flooring. The results on the tests on models of two bridge floors, one with a stringer spacing of two feet and the other with one-foot spacing are given. It is shown that battledeck flooring under a concentrated wheel load acts as an integral unit, distributing the load over several stringers, which act as simple beams. In the interior stringers it was found that the width-depth ratio of the plate acting as the compression flange of a T-beam may be regarded as high as 34. The model was tested with dead weights, and the results were computed from the strain and slope curves.

2. HISTORICAL FOREWORD

One of the most important problems in the design of battledeck floor systems is the action of the flat plate when subjected to uniform and concentrated loads. This problem is one of the oldest in applied mechanics, and one on which a good deal of work has been done, although as yet no complete solution has been found.

The flat plate problem was attacked first for the benefit of the musician and not for the engineer. Interest in the relationships between the pitch of a membrane and the area, pressure and thickness, led to a great many philosophical speculations on the subject, and a substantial prize was offered to the mathematician who would solve the problem. Many famous men worked on it, and although they did not solve it, they added greatly to the knowledge of plate theory. Kirchoff, Lagrange, Poisson, Cauchy, Navier, and Saint-Venant are only a few of the great names associated with the problem, and the prevailing theory of the thin flat plate, is the Kirchoff theory which has been put in its present form by Saint-Venant.

This theory lends itself to a fairly easy solution in the case of circular plates, symmetrically loaded. In recent years various solutions of much more practical problems have been offered. A. and L. Föppl[1°] offer many approximate and exact solutions of practical plate problems. Nadai[2], also submitted a number of solutions for various classes of plates and loadings, and this is probably the most complete set which had been offered up to that time. In this country, H. M. Westergaard[3], solves the problems of the medium thick slab, by using Fourier series to express the boundary conditions. This latter solution is particularly applicable to reinforced concrete slabs.

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° These numbers refer to references given in the Bibliography of this report

Marcus[4] solves flat plates by the elastic web method. He offers solutions for practically all types of plates and loadings. The method requires the solution of linear simultaneous equations.

Very little literature seems to be available, on the action of floor systems as a whole. The Bureau of Standards, in cooperation with the American Institute of Steel Construction has published reports on the Robertson Keystone Floor[5] and Battledeck Floors[6], under uniform loads.

In the case of the battledeck floor, it was found that the floor behaved as a unit, and that the plate on top the floor could be regarded as acting with the stringers in the form of a T-beam. No literature has been found for the problem studied in this investigation, namely, the battledeck floor under concentrated loads.

3. INTRODUCTION

A. General Information - The American Institute of Steel Construction sponsored this investigation into the action of concentrated loads upon battledeck flooring in order to obtain information upon the stresses and deflections in such flooring, for design purposes.

In order to further this subject a research fellowship supported by the Institute was established at the Fritz Engineering Laboratory of Lehigh University. The Institute provided the salary for the Research Fellow, the McClintic-Marshall Corporation furnished the steel in fabricated condition, and Lehigh University carried all other expenses. The investigation is carried out under the guidance of the Technical Research Committee of the American Institute of Steel Construction, of which Mr. Aubrey Weymouth is the Chairman, and the Messrs. H.G.Balcom, F.H.Frankland, O.E.Hovey, Jonathan Jones, and J.R.Lambert are the members. Acknowledgment is due all members of this Committee for their active interest in the work and their advice and guidance. Acknowledgment is also due Mr. E.L.Durkee and other members of the McClintic-Marshall Corporation for constructive criticism and valuable suggestions during the carrying out of the investigation, and to Mr. H.J.Bowles, Welding Supervisor, Bethlehem Steel Company, for advice and assistance in the construction of the modél, and to all members of the laboratory staff for their assistance during the construction and testing of the model.

The investigation has been carried out by experiments on two one-third size models. One model is based upon a prototype consisting of a bridge of two stringer panels 20 ft. long and 10 ft. wide, with stringers 2 ft. apart, and a 3/8-in. plate welded to the stringers with 4-in. beads, every 12 in. center to center. The second model is the same as the first, but the stringers are spaced 1 ft. apart in the prototype instead of 2 ft. A picture of the first model and loading rig is shown in Fig. 1. The big blueprint at the end of this report shows the details of the first model. The second model is the same as the first, but the stringers are spaced at 4-in. instead of 8 in., corresponding to an actual spacing of stringers of 12 in. A photograph of the second model which shows the stringers is shown in Fig. 2.

4. THE THEORY OF FLAT PLATES

The Kirchoff theory of the flat plate lies on several basic assumptions which are:

1. The thickness of the plate is small with respect to the span, so that the elongation of a filament parallel to the plane of the plate is proportional to its distance from the midsurface or neutral plane.

2. The deflection is small in proportion to the thickness of the plate.

3. There is no extension of the middle surface.

4. A straight line through the plate before bending remains straight after loading. This corresponds to the assumption in the beam theory that plane surfaces remain plane.

5. There is no normal traction across planes parallel to the middle surface.

6. Finally we must assume that the material behaves elastically, since our theory is based on the assumption that stresses are proportional to the corresponding strains.

This last assumption seems to be the one most likely to fall down when applied to flat plates, since even at very small loads, there seem to be small local yieldings and a consequent redistribution of stress. It is in this respect a safety factor, but it will throw the results of a theory, based on elastic behavior, in error.

The assumption that there is no extension of the middle fibre is not much in error, when the deflection is less than one tenth the thickness. This error is larger in the case of a fixed plate as the middle fibre must be extended, if we are to have any deflection, and for this reason also a fixed plate is stronger than a plate, simply supported.

The Kirchoff theory is mathematical, and has for its result, differential equations which must be solved to get a practical solution. These equations have been solved exactly for circular plates symmetrically loaded, and approximately for some rectangular plates by means of converging series.

Bryan in the Transactions, I. N. A. 1894, gives a fairly simple exposition of this theory.

He subdivides the stress in a plate as being of two types, one extensional, and the other flexural. The plate problem then falls into one of three classes; extension alone, flexure along, or a combination of the two.

A. Relation Between Stress And Strain In A Stretched Plate

Let μ = Poisoon's ratio
A = area of end of section
P = force
$\frac{P}{A}$ = tension under the force P

1. The section increases in length by $\frac{P}{EA}$ of itself.
2. The section decreases in width by $\frac{P\mu}{EA}$ of itself.

In Fig. 3 is shown the middle surface of a rectangular plate of thickness 2h.

The surface of the plate has no stress. Then the tension across AB per unit length = $\frac{P'}{2h}$.

This tension P', produces:

1. An elongation along BC of $\frac{P'}{2hE}$
2. A contraction along AB of $\frac{P\mu}{2hE}$
3. A contraction of the thickness of $\frac{P\mu}{2hE}$ of itself.

Combining strains, if ε_1 and ε_2 are the elongations of BC and AB:

$$\varepsilon_1 = \frac{P'}{2hE} - \frac{Q'\mu}{2hE} \qquad \varepsilon_2 = \frac{Q'}{2hE} - \frac{P\mu}{2hE}$$

Solving for P and Q:

$$P = \frac{2hE}{1-\mu^2}(\varepsilon_1+\mu\varepsilon_2) \tag{1}$$

$$Q = \frac{2hE}{1-\mu^2}(\varepsilon_2+\mu\varepsilon_1) \tag{2}$$

While the total amount the plate contracts

$$= 2h\left(\frac{P\mu}{2hE} + \frac{Q\mu}{2hE}\right) = \frac{(P+Q)\mu}{E}$$

$$= 2h\left(\frac{\mu}{1-\mu}\right)(\varepsilon_1+\varepsilon_2) \tag{3}$$

If the sides of ABCD are not parallel to the directions of the principal stresses in the plate, there will be in addition, equal shearing stresses, applied along the edges ABCD. Let their amount equal q' per unit length of edge. This will be $\frac{q'}{2h}$ per unit area of the faces A'B'B"A", etc. Also the figure ABCD will no longer be a rectangle after strain, but the angles differ from right angles, by a small amount, which we will denote by ϕ.

The former stress-strain relations hold good and in addition $q' = 2hG\phi = \frac{hE\phi}{1+\mu}$ where G denotes the simple rigidity of the substance (shear modulus) and equals $\frac{E}{2(1+\mu)}$

B. <u>Flexural Stresses</u> - In Fig. 4 is seen a section of a plate which is bent into the form of a cylinder. If ρ = the radius, the strain in a tangential direction PX = $\frac{z}{\rho}$, and the strain along PY parallel to the axis of the cylinder = 0. There is no stress in the radial direction PO'.

If P, Q, R, are principal stresses, ε_x, ε_y, ε_o, are principal strains at P, in directions PX, PY, PO, we have:

$$\varepsilon_x = \frac{z}{\rho}, \qquad \varepsilon_y = 0, \qquad R = 0$$

the first two give:

$$\frac{P}{E} - \frac{Q\mu}{E} = \frac{z}{\rho}, \qquad \frac{Q}{E} - \frac{P\mu}{E} = 0$$

whence the stresses at P' are given by:

$$P = \frac{E}{1-\mu^2}\frac{z}{\rho}, \qquad Q = \frac{E\mu}{1-\mu^2}\frac{z}{\rho}, \qquad R = 0$$

and from the condition of the stretched plate or from equation (3), it is readily inferred that:

$$\varepsilon_o = -\frac{\mu(P+Q)}{E} = -\frac{\mu}{1-\mu}\frac{z}{\rho}$$

whence the three strains are given by:

$$\varepsilon_x = \frac{z}{\rho}, \qquad \varepsilon_y = 0, \qquad \varepsilon_o = -\frac{\mu}{1-\mu}\frac{z}{\rho}$$

The strains and stresses are proportional to <u>z</u> and therefore greatest at the surface of the plate.

If the plate is bent into any other form, whose radii of curvature are $\rho 1$ and $\rho 2$, we would use the principle of superposition and add the strains and stresses due to the two radii.

$$\text{Then } P = \frac{Ez}{1-\mu^2}\left(\frac{1}{\rho_1}+\frac{\mu}{\rho_2}\right), \qquad Q = \frac{Ez}{1-\mu^2}\left(\frac{1}{\rho_2}+\frac{\mu}{\rho_1}\right), \qquad R=0$$

$$\varepsilon_x = \frac{z}{\rho_1}, \qquad \varepsilon_y = \frac{z}{\rho_2}, \qquad \varepsilon_0 = -\frac{\mu z}{1-\mu}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)$$

If ABCD is a small rectangular element of the plate whose sides are lines of curvature, it may be deduced by taking moments about AB, BC, that the action across A'B'B"A", gives rise to a couple whose axis is AB, and whose moment per unit length of side

$$AB = \frac{2}{3}h^3\left(\frac{E}{1-\mu^2}\right)\left(\frac{1}{\rho_1}+\frac{\mu}{\rho_2}\right)$$

and the action across the section B'C'C"B" gives a couple about BC,

$$\text{which } = \frac{2}{3}h^3\left(\frac{E}{1-\mu^2}\right)\left(\frac{1}{\rho_2}+\frac{\mu}{\rho_1}\right)$$

Since $\frac{2}{3}h^3 = \underline{I}$ per unit length, the bending moments about the lines of principal curvature are:

$$M_1 = \frac{EI}{1-\mu^2}\left(\frac{1}{\rho_1}+\frac{\mu}{\rho_2}\right) \qquad M_2 = \frac{EI}{1-\mu^2}\left(\frac{1}{\rho_2}+\frac{\mu}{\rho_1}\right)$$

The lines of principal curvature on the middle surface of the plate cross at right angles, but in rectangular plates, they are not always parallel to the edges, as might be supposed.

The principal curvatures expressed above are equal to:

$\frac{1}{\rho_1}+\frac{1}{\rho_2} = \frac{\delta^2\omega}{\delta x^2}+\frac{\delta^2\omega}{\delta y^2}$, where ω is the deflection of the plate.

A more general equation, which does not depend on the principal curvatures, and which can be derived from the above by the principles of elasticity is:

$$\frac{EI}{1-\mu^2}\left[\frac{\delta^4\omega}{\delta x^4}+\frac{2\delta^4\omega}{\delta x^2\delta y^2}+\frac{\delta^4\omega}{\delta y^4}\right] = z$$

where Z is the load per unit area.

A solution of the above equations, considering bending and stretching, if we can solve the differential equations, will give us the desired results.

The Kirchoff theory as such, does not take into account the extensional stresses which are derived in the first part of Bryan's treatment, and they should be taken into account when the deflection is appreciable.

The portion of the extensional stresses which has a resultant, at the edge of the plate will be called the catenary stresses. The part which is in equilibrium within the plate, may be called secondary stretching stresses, though their magnitude may be of the order of the other stresses.

The secondary stresses have no external resultant, and thus are not directly produced by the applied loads. Their effect on the safety of the structure is entirely subordinate to that of the primary stress. If the material is ductile, a redistribution of secondary stresses can occur which generally renders them of small importance. It is necessary that the deformation for redistribution should be small. In the case of a flat plate, fixed at its edges, as the load is increased, the bending stresses increase and at the edges where there is local stress concentration, we have a slight yielding, the middle fibre stretches a little, taking up stress, and we have a redistribution of stress with very little deformation. After the redistribution, the unit stress will quite often be less than it was originally.

5. <u>THE THEORY OF MULTIPLE STRINGERS</u>

The distribution of a wheel load between the various stringers of a bridge floor has always been a moot point. Very little literature seems to be available on the subject, and it is hoped that this research will throw some light on the matter, particularly in the case of battledeck floors.

In a battledeck floor, as the load is applied to a stringer, the stringer naturally deflects. As it deflects, the plate begins to act as a cantilever beam to transmit shear to the next stringer. This will cause the next stringer to deflect, and the distribution goes on till it eventually reaches the exterior stringer. The amount of shear which a stringer will transmit will depend on the relative deflection between any two stringers. Since the deflections vary as the cube of the spacing between the stringers and inversely as the cube of the thickness, for a floor having equal spacings between the stringers, there should be a constant proportion of the shear transmitted between each stringer. For a flexible floor this constant will be small. As the floor is stiffened, the constant will become larger and the loading will be spread much more appreciably over the stringers. In discussing the experimental results, attempts have been made to establish definite ratios between the amount of load supported by each stringer of the model.

6. <u>OUTLINE OF TEST PROGRAM</u>

A. <u>General</u> - It was proposed to study in this program the problems connected with:

1. Battledeck flooring for highway bridges.
2. Battledeck flooring under concentrated loads.

These problems require, for their solution, the determination of:

a. The strength of battledeck flooring under concentrated loads.
b. The deflection of battledeck flooring under concentrated loads.
c. The properties of the floor plate in distributing the load over various stringers.
d. The amount of floor plate acting with the stringers as the compression flange of a T-beam.
e. How far along the longitudinal axis of the plate is the effect of the concentrated load noticed?
f. The effect of changing the distance between the stringers.
g. The fixity of the ends of the stringers.
h. The fixity of the ends of the plate.

The program was carried out by making tests on a one-third size model, varying the loads and span of the stringers, and the conclusions are drawn from the results of these tests. Eventually it is planned to run a test on a full size floor panel as a check on the model results.

This program was essentially that which was outlined by the Technical Research Committee of the American Institute of Steel Construction at its meeting in New York City on October 1, 1934.

B. Design of Model - The model was based on one lane of a bridge panel, 20 ft. long and 10 ft. wide. The floor of the prototype consisted of a 3/8-in. plate resting on stringers 24 in. apart. This floor was designed for a H-20 loading.

The stringers were assumed to act as fixed end T-beams, but the stringers were designed to take a full wheel load on the assumption that the plate was so thin that it could not carry any appreciable shear to the next T-beam, until the stringer had deformed more than desirable. These assumptions are also equivalent to assuming that the stringers are free ended, and must be designed for one-half a wheel load.

C. Required Section Modulus:

$$\text{Live Load} \quad S = (22{,}400)\left(\frac{20}{8}\right)\left(\frac{12}{18{,}000}\right) = 37.3 \text{ in}^3$$

$$\text{Dead Load} \quad S = \frac{(29.3)(2)(20)^2(12)}{(12)(18{,}000)} = \frac{1.3\text{in}^3}{38.6\text{in}^3}$$

This requires a 12-in. W.F. 28-lb. I-section, flange = 6.5 in. and web = 0.24 in. This beam with a 3.8-in. plate 2 ft. wide on top of it has a section modulus of 40.96 in^3.

This conception of the bridge then involves: transverse floor beams 20 ft. center to center; longitudinal stringers, 12 in. W.F. 28-lb. beams, 24 in. center to center, and 3/8-in. floor plate; the stringers being coped flush with the floor beams so that the floor plate can be welded to both.

The model was designed to give stress conditions similar to those of the prototype. Each panel was made one-third size, and was 80 by 40 in. The model consists of two panels. The plate was made 1/8-in. thick. The stringers were 3-in. 5.7-lb. I-beams whose flanges were cut down to a width of 2.16 in., to make it one-third the 6.5 in. of the prototype. This gave the T-beam in the model a section modulus of 1.83 in^3., whereas it should be 1.54 in^3. This is stiffer than the model should be, but as close as could be obtained without a lot of expensive machining.

For the floor beam in the model, a standard 4-in. 7.7-lb. beam was used. This has an $\underline{S}$ of 3.0 in^3, whereas 2.6 in^3 was called for, but the floor beam should have little effect on the results.

The model was welded by manual arc welding using a 1/8-in. bare electrode. The plate was butt welded over the floor beams with continuous welds, and also welded continuously along the exterior stringers. The interior stringers were welded to the plate by intermittent welds 1-1/4 in. long about every 4 in. Great care had to be taken in welding to prevent warping and shrinkage, and on the first attempt, the floor which seemed apparently all right before lunch time, warped 4 in. on cooling over the noon hour, and the plate had to be ripped off and another one put on.

The material in the floor was ordinary grade structural steel though the plate showed a very low yield point, and ultimste strength. The result of a typical sample is shown in Fig. 5.

Sample specimens were taken from the plate in various places, including positions adjacent to the burnt edge of the plate and adjacent to welds. The physical properties for the plate close to the weld, seemed slightly lower than the ones further away, but there was not enough difference to draw any conclusions.

The model was held in a frame consisting of vertical posts made up of 8-in. channels, and the channels were braced with angles. The floor beams of the model were welded to the vertical channels to simulate the connections of the floor beams to the hangers in an actual bridge.

The second model was exactly the same as the first, but the stringers were spaced at 4-in. intervals instead of the 8 in. in the first model. Actually all the stringers were in from the beginning, but in the first model, half of them hung loosely, and could not take any load. The model was assembled this way because there would not have been enough room to fit the stringers for the second model in place after the first ones were welded in. After the completion of the tests of the first model the additional stringers were welded into place.

Local stresses were introduced in the plate from the welds, and the first time load was applied, there was a large number of local yieldings even under very small load. Due to these yieldings the results of the first run were very erratic.

Before welding in the extra stringers for the second model, strain readings were taken on all the stringers and these were repeated after the welding. Fairly large welding stresses were noted. These stresses were somewhat larger at the center of the beams than they were at the ends, and were compressions in the original beams, while they caused tension in the beams which were newly welded in.

The maximum stresses for each stringer are shown in Table I. These stresses were not local, but continuous and since they were all measured on the tension side of the beams, the tension values are high enough to cut down our safety factor a good deal. These stresses were present, in spite of the fact that every precaution possible was taken in order to keep the welding and cooling stresses at a minimum.

C. Loading Rig - The original panel was designed for a H-20 loading. The U. S. Bureau of Public Roads has approved a loading area of 20 by 10 in. for the rear wheel of such a loading. This assumes a rear wheel with a tire 20 in. wide, and with a tire pressure of 112 lb per sq in., so that 10 in. of tire bear longitudinally.

The model was loaded by dead weights on a loading rig made of a cast iron block to which a frame was fastened to carry the additional dead load. The initial weight of the rig was 400 lb. Additional load was applied by adding 50-lb. blocks. A photograph of the loading rig is shown in Fig. 6. The load is applied through a steel bearing plate which is one-third size or 6-2/3 by 3-1/3 in. Under the bearing plate was placed a piece of soft rubber 1-1/2 in. thick to keep the load constantly uniform as the plate deflected, and a piece of cellotex was placed under the rubber to keep the area in contact with the plate constant.

A load of 2489 lb. should cause the same stresses in the model which the rear wheel of a H-20 truck would cause in the full size bridge panel. The model was loaded up to 2500 lb., and readings of stress, slope, and deflection were taken at all necessary points.

In order to show that the loading rig would give a uniform load distribution beneath it, and in order to compare the results of the stress distribution and that of a tire, a comparative load test was made. The floor was loaded through the rig, and strain measurements were taken around the load at the points shown in Fig. 7. Then the floor was again loaded through a tire, as shown in the photograph of Fig. 8. The results of these loadings are compared in Table II, and it is seen that they agree within the limits of experimental error.

This indicates that the loading rig gives a fairly uniform load, as is shown particularly by the agreement at gage points 4 and 5, which are the significant points in the comparison, since they are directly under the load. The high value for gage point 5 would be regarded with suspicion except for the fact that the strain gage recovered completely upon removal of the load, showing elastic behavior at this point.

D. Observations and Instruments - In a regular run, strain measurements were taken along the longitudinal axis of the stringers with a 10-in. Whittemore strain gage. Slope measurements were also taken with a level bar of 5-in. gage length. Measurements were taken transversely between the stringers with an 8-in. Berry gage, in order to determine the rotations of the stringers. The load was then applied and all the readings were repeated. The temperature of the laboratory was kept as nearly constant as possible, and in most of the runs was held within a range of four degrees Fahrenheit at the most. In some of the runs this was impossible, and there was a variation of temperature up to twelve degrees Fahrenheit. Corrections were applied to the strain readings for any temperature variations from the readings on an invar steel and a mild steel standard.

A total of about sixteen complete regular runs were made on the two models, applying the load in various positions on the floor.

Several other minor tests were made. Rows of Ames dials were placed beneath the stringers and plate to measure the deflections and to check the slope readings.

The plate stresses were measured at important points with Huggenberger tensometers and also with a 2-in. Olsen strain gage.

The level bar was made, so that various gage lengths from 1 to 6 in. could be used. However, the 5-in. gage length was used practically exclusively. It was fitted with a very sensitive bubble so that readings could be repeated to 0.0001 in., if the point hit the same spot, but hitting the exact spot was practically impossible, and so the accuracy was limited by the irregularities in the plate surface. Generally, readings could be repeated to within 0.0008-in.

The Whittemore strain gage was of the fulcrum type, and fitted with a 0.0001-in. Ames dial. It is accurate to within 600 lb per sq in., as a tolerance of 0.0002 is allowed in repeating a reading.

The 2-in. Olsen gage is equipped with a Last-Word Dial, but cannot be relied upon, perhaps, to within 1000 lb per sq in. The Huggenberger tensometers are accurate to within 300 lb per sq in.

E. Computations - From the data as taken, slope curves were plotted. These were differentiated twice in order to get the moment and shear curves. This was done by applying an empirical equation to the slope curve for each stringer, which fitted it as accurately as possible, and differentiating the equation numerically. This gave the results for moment and shear in terms of EI, and by multiplying these values by the proper values of E and I, to give the actual known shears, we had a means of evaluating I. The strain readings would give a check on the results.

The measured stresses plotted as straight lines along the stringers, showing that the load acted as point loading. This showed that the slope curves were second degree curves, and the computations were based on this assumption.

A typical stress diagram for a stringer is shown in Fig. 9, and a typical slope diagram is shown in Fig. 10.

The small values for slope and stress in the second panel, show that practically no load is carried over to it, and the effect of the second panel can be practically disregarded.

7. TESTS OF MODEL OF 24-INCH STRINGER SPACING

A series of nine runs, as described in the test program, were made on the first model. The positions of the load in these various runs and the nomenclature of the stringers and floor beams are illustrated by Fig. 11.

In Series 16, the load was placed between stringers E and G, in the middle of the first panel. In Table III, are shown the results of the test. It is seen that the two adjacent stringers E and G, take eighty per cent of the load, while the two next ones take about ten per cent. It is also seen that the stringers, which are away from the direct influence of the load, carry over about 25 per cent of their load to the adjacent stringer. The beams act as if they were partially fixed, the average fixity factor on the left being about 15 per cent and that on the right being about 39 per cent. Due to this fact, the left shear is less than the right. These fixity factors are the ratio of the moments at the supports to those of a fixed end beam.

In order that the shears should equal the applied load, it was found that the moment of inertia of a stringer had to be 3.54 in^4. This required that 5-1/2 in. of plate act in the compression flange. With these values, the computed center moment was found from the slope curves, and the fibre stresses were computed. The measured stresses do not agree with the computed as well as they should, and the reason for this will be discussed later.

The center deflection was computed from the formula, $d = \frac{Wl^3}{192EI}(-4+2K_1+K_2)$. This formula was derived from moment area theorems, and while slightly approximate is good within two per cent, if the fixity factors do not differ by more than fifty per cent.

The deflection from the slope readings was obtained by summing up the areas underneath the slope curve. The agreement between the observed and measured deflections, is a check upon the accuracy of the differentiation and the computations.

In Table IV, are shown the results of Series 17A, on the first model. In this run the load was placed directly on Stringer E at its center. However, the loading area is so wide that the bearing block lapped over into the plate so some of the load was transferred directly through the plate to C and G. Again the ratio between the shear carried from one stringer to another, in those away from the load, is seen to be 0.24. The average left fixity factor is 23 per cent and right fixity factor is 35 per cent.

The value of the moment of inertia of the stringers to make the shear equal the load was 3.35 in^4. for the interior stringers, and 2.84 for the exterior stringer A. This corresponds to 4-1/4 and 1-7/8 in. of plate in T-beam action respectively. The values for I were worked out as follows: First the shears were computed using a value of I corresponding to full T-beam action, that is, using 8 in. of plate for the top part of the T in the interior stringers, and 4 in. for the exterior stringers. These values made the shear come out too large, so next the values for the moment of inertia were multiplied by a factor so the shears would equal the load, and then the plate required for this value of the moment of inertia was computed.

The left reactions are again found to be less than the right, as would be expected, since the left fixity factor is the smaller. It is seen that Stringer E directly beneath the load takes fully fifty per cent of the load.

The center moment, which is also the maximum moment, and the stresses have been computed as in the previous series. The agreement between the deflections, both computed and measured, gives us a check on the work.

Series 19, is the same as 17A, but the load is placed on Stringer C, instead of on E. This places it, on a stringer which is next to the exterior stringer, and consequently the load cannot be distributed so widely. Thus Stringer C carries 55 per cent of the load. The amount of plate in T-beam action was found to be the same as in Series 17A, namely 4-1/4 and 1-7/8 in.

The left fixity factor averages 20 per cent, while the right fixity factor is 37 per cent. The center stress, moments, and the deflections were found as in the previous cases. Again, the computed and measured deflections check the work.

In Series 20, the load was placed between Stringers A and C, and the results are shown in Table VI. The effect of T-beam action is much less marked in this case, and we have but 1/2-in. acting with the exterior stringer and 2 in. with the interior stringers. Stringers A and C, carry ninety per cent of the load between them. Stringer C carries over about 0.22 its shear to Stringer E.

The left fixity factor is 33 per cent and the right fixity factor 40 per cent. The moments, stresses, and deflections were found as before and agree quite well.

Series 21, shows the worst loading condition for the floor. Here the load was placed on the center of Stringer A, with the long edge of the bearing block, bearing longitudinally along the floor. This is equivalent to having a truck back up against the curb of a bridge. Stringer A took 65 per cent of the load.

The amount of plate in T-beam action is found to be the same as in Series 20. Stringer C carries over 25 per cent of its load to Stringer E. The average left fixity factor is 39 per cent and the right fixity factor is 49 per cent. The measured stress in Stringer A gave values which would require much more plate in T-beam action than there actually was, which shows there were other stresses in the stringer besides bending stresses.

The center deflection does not check as well as it might for Stringer A, but it checks reasonably close for the other two stringers. We have a deflection of 0.149 in. under the load and this was the largest one measured. Thus this loading has both the maximum deflection and the maximum stress.

In Table VIII are given the results of Series 22. In this series, the load was placed between Stringers E and G, at the quarter-point as shown in Fig. 11. This loading is thus the same as Series 16, but the load is further along the stringer. The amount of plate in T-beam action is also 5-1/2 in. as in Series 16. The amount of shear transferred from one stringer to another is seen to be about 0.08 the shear in the stringer, instead of the 0.25 which we have in Series 16. This is readily explained when we think that the shear transferred from one stringer to another depends on the relative deflection of the stringers to each other, and since the stringer deflects less at the quarter point, the difference in deflection between two adjacent stringers is going to be less.

The average fixity factor of the left end is 27 per cent, while that on the right end is 16 per cent. The moment, shears, stresses and deflections were computed as in the previous cases.

Series 23, gave stresses so small that they were not worth while computing, as the stresses lay within the experimental errors of the instruments. Series 24 and 25 were similar in this respect.

In Series 24, however, the results for Floor Beam M were computed, and they are given in Table IX. The floor beam took all the load. In this case we had 16 in. of the plate acting in T-beam action. The large value for this case being, undoubtedly, due to the supporting and bracing action of the stringers on the plate.

The fixity factor worked out to be about zero, which was very surprising, for the floor beams were welded to the hanger posts or verticals with a bead all the way around the beam. The vertical was evidently much more flexible, relatively, than the floor beam.

Several supplementary tests were run on this model. In one of these, rows of Ames dials were placed below the various stringers to read the deflections as the load was applied.

In Fig. 12, the deflection of Stringer E is plotted against the load. The load was placed in a position corresponding to Series 16. It is seen that the plot for the stringer is a straight line which is as would be expected, and it also shows that the amount of plate acting as a T-beam is constant, and does not vary with the load.

A plot of the center deflection of the plate is given in the same figure. This curve slopes upward to the left showing that as the plate becomes loaded, it takes more load to give equal increments of deflection. In other words, we have catenary action helping to support the plate.

In Fig. 13, the deflections of the plate and Stringer E are plotted against the longitudinal axis of the stringer for a 2500-lb. load. The deflection curves for quarter-point loading are also given. In the case of the stringers, we have a slight initial reverse curvature of the deflections at the ends of the stringers. This shows slight partial fixity.

The curve for the plate in sharp contrast to that of the stringers slopes gradually and then bellys down quite sharply underneath the load. This was quite characteristic of the plate, to have a large deflection directly beneath the load, and no matter how small the load, there was always a small permanent set after the first time a point on the plate had been loaded.

In order to supplement the tests on the floor, tests were made on various sizes of plates welded to several short H-beams. The span of the plates was 8 in. between the centers of the legs of the H-beam. The deflections of the plates at various points were measured. A picture of the set-up is shown in Fig. 15. In all cases, the deflection curve was similar to that shown for the plate in the floor, in Fig. 12. If the load was increased enough, however, this curvature was reversed, and the material yielded gradually. For 1/8-in. plate, on 8-in. spacing, this occurred at about 3500 lb. However, the ultimate load was 38,000 lb., and the plate finally failed in punching shear. In another similar test, in which the plate was welded to a lighter H-beam, so the sides gave more easily, the plate could not be broken, for when the ultimate load of 50,000 lb. of the machine was reached, all that had occurred was a very marked yield of the H-beam sideways so that the plate began to act like a buckle plate.

In these tests, there was quite a horizontal force in the plate as was evinced by the sidewise yielding of the H-beam. This was also true in the case of the floor model. The bottoms of the stringers rotated a good deal, and a plot of the rotation of one of the stringers is shown in Fig. 14. In some stringers, namely the exterior ones, the rotation was three times as much as that shown in the figure. However, the top of the stringer showed little or no rotation, which indicated that the stringers must rotate about the top of the plate. It is not readily seen what this rotation is due to. Partly it must be due to the fixity moments of the floor plate. If the tops of the stringers moved more, we could say it was also partly due to the horizontal force in the plate. Evidently the stringer takes some of the load in torsion.

The plate stress was also measured underneath the load, and in various critical points around it. The maximum stress was directly underneath the load, in the center, between the stringers. Its value was about 43,000 lb. It is interesting to note, that if the plate is figured as a fixed-end beam using the clear span of the plate, we will also get this value for the stress. The stress in the top of the plate, over the stringers, showed that we had practically one hundred per cent fixity at the edges of the plate.

8. TESTS OF MODEL OF 12-INCH STRINGER SPACING

The second model, as has been said before, is the same as the first, except that the stringers are four inches center to center.

The results on the second model are similar to those on the first. Due to the decreased stringer spacing, the floor is stiffer and thus the load is distributed over more stringers.

In Table X are shown the results of Series 32, in which series the load was placed directly on top of Stringer F. The positions of the load series, and the nomenclature of the stringers of the second model are shown in Fig. 16. The width of the loading area is such in Series 32 that the load rests directly on three stringers, E, F, and G. We have full T-beam action in this series. Stringer F takes the maximum portion of the load or about thirty per cent. The ratio of shear carried from one stringer to another in those away from the load is about 0.43, instead of 0.25 that we had in the first model.

The average left fixity factor is about 12 per cent, and the right fixity factor is about 35 per cent.

The center moment, stress, and deflection, were computed for the second model in the same manner as for the first. The slope readings check the deflection. However, the ratios between the various stringers do not stay quite so constant for the shear, moment, and deflection, as they did in the first model. This shows that the curves are not exact second-degree curves, although they are so assumed, and consequently, a slight error creeps in with the differentiation and integration.

In Series 33, the load is placed on top of Stringers E and F. Again, we have full T-beam action. Stringers E and F take 28 per cent and 26 per cent of the load respectively. The average left fixity factor is 27 per cent, and the average right fixity factor is 42 per cent. The center moments, stresses, and deflections were computed as in the preceding cases.

Series 34 is the same as Series 32, but the loading area is at right angles to what it was in Series 32, and consequently most of the load was taken by Stringer F, which has 38 per cent of the load. It is a more severe loading than Series 37. The ratio of the shear carried from one stringer to another is about 0.53. The average left fixity factor is 25 per cent and the average right fixity factor is 45 per cent. The center moment, stresses, and deflections were computed and they check as before.

Series 35 is the critical load for this model. The load was placed longitudinally along the exterior stringer. The ratio of shear carried from one stringer to another is here about fifty per cent for the average. However, it is the value of Stringer D which brings the average up to this amount, and since Stringer D has the smallest values observed in the test, they are not as reliable as the others.

In this case we do not have full T-beam action. For the interior stringers, we have 3 in. of plate acting in the top flange and 1-1/2 in. acting in the top flange of the exterior stringer. Stringer A takes slightly more than one-half the load or 53 per cent. The average left fixity factor is 18 per cent and the average right fixity factor is 52 per cent. The center moments, stresses and deflections were found the same as previously, and check quite well.

Supplementary runs were taken on this model also to measure the deflection of the stringers with Ames dials. Fig. 17, gives the deflection curves of the stringers. The relative deflection of each stringer gives the relative load carried by each stringer, with the exception of Stringer A, whose smaller moment of inertia will cause it to deflect a relatively larger amount. Unfortunately, the Ames dials could not be placed under the stringers during a regular run, for they would prevent reading the strains. However, the results of the dial runs check the slope readings remarkably well, considering that they were taken at different times.

A reading of the stress in the plate itself could be obtained only under the center of the load, in this model, due to the cramped space. For a 2500-lb. load, its value was 17,000 lb per sq in. The clear span between the stringers was 2.84 in. It would take a span of 3.28 in. to give us this stress, figuring the stress as a fixed end beam. This distance of 3.28 in. is incidentally the distance between the fillets of the stringers. Since the load covers the flanges, they deflect, and throw the supporting reactions of the plate further apart, and in this case the reactions of the plate can be considered as acting on the outside of the fillets.

The plate deflections were also measured in this model and the maximum plate deflection was 0.0175 in.

9. COMPARISON BETWEEN THEORY AND TEST RESULTS

There were not enough tests run to give any comparison between the results of the tests and the plate theory. What plate equations to use in the comparison would be another question. However, one thing seemed quite clear, the stresses in the plate were much less than those given by practically all plate stress equations, and the deflections were less. However, there was catenary action present in the plate, and practically all the theories disregard this effect.

For this ratio of width to length of the plate, the simple beam formula, by a judicious choice of the span length, gave the best results. The span was not varied enough in these tests to give any definite conclusion, but in those cases tested, the loaded distance between the rigid supports of the plate seem to be the one to use. For example, in the first model, the load extended just over the clear distance between the flanges and thus the edges of the flanges would be the supports for the plate. In the second case, however, the load extended all the way between the stringers and deflected the flanges of the I-beams so that the plate had to carry the reactions, practically to the web of the stringers.

High local stresses were measured around the load. Huggenberger tensometers were placed as close to the load as possible, and stresses from 20,000 to 35,000 lb per sq in. were noted. They caused local yieldings which left slight permanent deformations.

The load was distributed between the stringers in proportion to the relative deflection of the stringers, as was predicted by the theory. The constant factor was about 0.24 for the first model and about 0.46 for the second model.

The stresses in the stringers do not check the theory. The reason for this is that the stresses as figured are only the simple bending stresses of a fixed beam. Since the beams are welded to the floor beams, tensional stresses can occur, and also moments may be applied. There are four types of stresses which are added to the simple bending stresses along the tension flange. First, due to the tension in the beam, the bottom of the beam increases in length, this will put compression on the ends of the beam, and tend to reduce the tension. Second, due to the deflection of the beam, any longitudinal axis shortens, and causes tension along that axis. Third, due to the rotation of the bottom flange, the center of the bottom flange, along which we incidentally had our gage holes, shortened in length and caused tension in the gage line. Finally, these effects in any stringer would have a reaction on the floor beam, rotating the floor beam, which in turn would apply a couple and a tensional stress to the other stringers. An attempt was made to evaluate these stresses, and apply them as a correction, but so many assumptions had to be made, that they did not check any better before than afterwards.

The effect of these stresses is greatest on the stringer which is loaded. They reduce the compression and increase the tension in that stringer. The stringer has now a large effect on the rotation of the floor beam. The floor beam tends to rotate, and it rotates enough so that the secondary stresses, as these may be called, in the other floor beams are actually offset. In the stringers a distance away from the load, the tension is reduced, in some cases to zero, and the compression is increased. In between the two extremes, we find that the secondary stresses neutralize themselves, and our stresses are only bending stresses.

The rotation of the floor beam was measured in Series 32. It was found to amount to 8-1/2 minutes in this case. This varied along the axis of the floor beam, as the square of the distance from the loaded beam. The rotation was around the center line of the stringer, so that there is in effect a couple and a tension applied to the stringers, since the center line of the stringer does not coincide with the center of gravity axis of the T-beam.

For practical purposes, the secondary stress is important only in the stringer under the load, since that condition is what the stringer must be designed for. Here the tension which is critical in the design, is about 25 per cent more than computed.

10. SIGNIFICANCE OF RESULTS

The results of the tests on these two models, show that they were amply strong enough for H-20 loading. The first model had undesirable deflections in the plate, which left permanent set, and a surfacing, if it had been put on this model would undoubtedly, have broken up. The second model was all right in this respect. As a matter of fact the load could not be placed on the second model without having it rest on the stringers somehwere.

The load was carried by only a few stringers and the amount carried over from one stringer to another was so small that it died out in a short space. For example, the load carried by the fourth stringer away from the loaded stringer in the first floor would be only $(1/4)^4$ of the loaded stringers or $(1/2)(1/256) = 1/512$ the wheel load. Since the wheel loads are six feet apart in a H-20 loading, it is seen that a stringer has to be designed for one wheel load only.

The results of the model are not entirely uniform. The model was made of standard materials, and as carefully as possible. However, discrepancies of 1/8 to 1/16 in. could and did occur in welding up the stringers, and this would cause one stringer to take more than its proportionate share of the load from the next one. This practice would also occur in an actual structure. In spite of this, the results are quite consistant.

The amount of plate acting as the compression flange of a T-beam was variable, and depended on the position of the load and beam. In those cases where the load was between the stringers and would help to stiffen the flange, it may be considered as high as 5-1/2 in. Where the load is on top of an interior stringer, there is 4-1/4 in. of plate acting. However, when the load is on top of an exterior stringer, there seems to be hardly any plate acting. In the center floor beam 16 in. of plate were in action, a very large amount. The amount of plate acting in the flange varied with the stiffness of the plate and how it might be braced by the load or other stringers. In the case of the exterior stringers, if we had a large amount of plate acting, we would have an unsymmetrical beam.

Changing the distance between the stringers stiffened the floor immensely, and distributed the load over more stringers.

The average of all the fixity factors on the left was 25 per cent and that on the right 39 per cent. These values are so small and the fixity factor is so variable, that it cannot be taken into effect in any computations. The value for the right end is higher since the second panel has a stiffening effect. The reason for the low fixity seems to be due to the rotation of the floor beams and the rotation in the connection between the floor beam, and the web angles of the stringer.

The plate acted as fixed and may be regarded as such. This will cut down the maximum bending stresses in the plate. The plate stresses seemed very high in some cases, but no harmful effects were noted, and they recovered in most cases.

11. RECOMMENDED METHOD OF DESIGN

From the results of these tests, it is recommended that battledeck flooring be designed on the following basis.

A. Plate:- As a fixed end beam, whose span is the length between the rigid supports of the loaded plate.

B. Stringers: 1. Interior. As simple beams strong enough to carry one half the wheel load.

As T-beams, with as high as 12 in. of plate in compression if the plate is 3/8-in. plate.

It is recommended that a wide flange beam be used if possible, for the stringers, since it decreases the span of the plate and consequently the plate stress, and it makes it easier to cope the stringers in the floor-beams giving a shallower floor.

It might also seem desirable to increase the thickness of the plate and the span. If the plate thickness were doubled, the span could be practically doubled, and the clear span would be more than doubled, since we would have one less flange width for every two spans. We would increase the weight of the plate 5.7 lb per sq ft. for each extra 1/8 in. of thickness. If we have a 3/8-in. plate on 12-in. W.F. 28-lb. beams, 1 ft. apart, as in our second model, and change the stringer spacing to 2 ft., we will have to increase the weight of the beam only slightly, and cut out every other stringer, while we increase the weight of the plate 17.1 lb per ft. This gives us a net decrease of about 6 lb per ft. More study and experimentation on different sized plates is necessary before any rigid conclusions can be drawn.

C. Floor Beams:- The floor beams may be designed as a simple beams with as much as 48 in. of 3/8-in. plate acting in the top flange, when the stringers are coped to the floor beam, and the plate welded to both the stringers and floor beam.

12. SUMMARY

From the tests on these two models, the following conclusions are drawn for similar cases:

1. Large inherent welding stresses, are to be found in the stringers of battledeck flooring.

2. Great care is required, in welding, to minimize these stresses.

3. The loading rig as designed in the model gave a uniform stress over the bearing area, and it gave the same stresses as when the same load was applied through a tire, showing that the tire gives a stress distribution which is sensibly uniform.

4. The stresses and deflection in the beams may be computed by the ordinary beam formulas, keeping in mind that secondary stresses will increase the maximum tension about 25 per cent.

5. The plate acts as a T-beam in the interior stringers, the ratio of the width of the plate to its thickness acting as such can be safely considered as high as 34.

6. The amount of plate in T-beam action does not vary with the load.

7. Very little T-beam action can be counted upon in the exterior stringers.

8. The stringers can not be taken as fixed end beams and should be regarded as simple beams. Any slight fixity which is present can be regarded as a small safety factor.

9. The stringers distribute the load in proportion to the relative deflection between the stringers. Thus the distribution is the greatest when the load is in the center of the panel, and it decreases as the load approaches the floor beams. The distribution factor varied as the thickness of the plate and the distance between the stringers.

10. The effect of the concentrated load on the plate is quite local.

11. Decreasing the distance between the stringers, stiffens the floor, and increases the load distribution. However, the maximum load on any one stringer is reduced only slightly.

12. The plate can be regarded as fixed at the edges of the stringers.

13. The plate has a very large reserve strength ratio, over its initial yield point, due to the catenary action which is induced as the plate deflects.

14. In the models, the deflection of the plate was so large with respect to its thickness that all ordinary plate theories do not apply.

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TABLE I - MAXIMUM WELDING STRESSES

Stringer	
A	-1200
B	+4500
C	-2400
D	+6000
E	-4200
F	+6300
G	-3900
H	+6900
I	-1800
J	+5700
K	-1800
Floor Beam	
L	-3900
M	-6600
N	-6300

TABLE II

COMPARISON OF RESULTS - TIRE VERSUS LOADING RIG

Point		Tire	Loading Rig
		lb per sq in	lb per sq in
C_4	(Top)	- 1,800	- 1,800
D_4		- 2,100	- 2,400
H_4		- 2,400	- 2,400
I_4		- 1,800	- 2,100
C_4	(Bottom)	+ 300	+ 600
E_4		+ 9,000	+ 9,600
G_4		+10,500	+10,800
I_4		+ 600	+ 600
H_1	(Top)	- 500	- 2,300
H_2		- 900	- 900
H_3		- 1,800	- 1,800
D_1		- 1,800	- 2,300
D_2		- 500	- 1,800
D_3		- 1,800	- 4,200
4	(Bottom)	+20,200	+17,000
5		+41,300	+43,200
		in.	in.
2_4	(Rotations)	0.0368	0.0371
6_4		.0665	.0671
4_2		.0353	.0342
4_6		.0335	.0342
5_2		.0356	.0354
5_6		.0368	.0374

TABLE III

SERIES 16 - FIRST MODEL

Str.*	Shear		Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor		Computed Center Moment	Ratio to Next Str.*	Center Stress		Center Defl.		Ratio to Next Str.*
	Left	Right				Left K_1	Right K_2			Computed	Measured	Com-puted	From Slope Reads.	
						per cent		in-lb		lb per sq in		inches		
C	101	119	220	0.21	8.8	13	42	3,790	0.21	+ 2,260 - 1,310	0 - 1,500	0.0185	0.0178	0.23
E	507	555	1062	1.00	42.6	18	34	18,500	1.00	+10,300 - 6,030	+ 9,600 - 2,300	.0880	.0769	1.00
G	470	488	958	1.00	38.3	15	26	17,250	1.00	+ 9,640 - 5,620	+10,150 - 2,250	.0828	.0854	1.00
I	122	138	260	0.27	10.4	11	55	4,330	0.25	+ 2,430 - 1,400	+ 800 - 1,800	.0211	.0205	0.24
Total	1200	1300	2500		100.1									

* Stringer

Properties of Section Used in Computation:

Plate in T-Beam Action -- 5-1/2 inches

$I = 3.54\ in^4$

$S = 1.79\ in^3$ and $3.07\ in^3$

TABLE IV

SERIES 17A - FIRST MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1	Fixity Factor Right K_2	Computed Center Moment	Ratio to Next Str.*	Center Stress Computed	Center Stress Measured	Center Defl. Computed	Center Defl. From Slope Reads.	Ratio to Next Str.*
						per cent		in-lb		lb per sq in		inches		
A	61	75	136	0.23	5.4	27	29	2,290	0.25	+ 1,140 - 1,370	0 - 2,500	0.0135	0.0129	0.30
C	265	315	580	0.47	23.2	38	42	9,320	0.43	+ 5,270 - 3,430	+ 5,250 - 2,600	.0434	.0426	0.41
E	538	699	1237	1.00	49.5	11	35	21,600	1.00	+12,200 - 7,940	+15,400 - 4,000	.1120	.1075	1.00
G	201	241	442	0.38	17.7	15	27	7,940	0.37	+ 4,490 - 2,920	+ 4,500 - 3,100	.0402	.0386	0.36
I	47	58	105	0.24	4.2	23	43	1,740	0.22	+ 980 - 640	+ 800 - 900	.0087	.0083	0.22
Total	1112	1388	2500		100.0									

Properties of Sections Used in Computations:
Plate in T-Beam Action -- 4-1/4 inches
$I = 3.35 \text{ in}^4$
$S = 1.77 \text{ in}^3$ and 2.72 in^3

Exterior Stringer A
Plate in T-Beam Action -- 1-7/8 inches
$I = 2.84 \text{ in}^4$
$S = 1.67 \text{ in}^3$ and 2.00 in^3

* Stringer

TABLE V

SERIES 19 - FIRST MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1	Fixity Factor Right K_2	Computed Center Moment	Ratio to Next Str.*	Center Stress Computed	Center Stress Measured	Center Defl. Computed	Center Defl. From Slope Reads.	Ratio to Next Str.*
						per	cent	in-lb		lb per	sq in	inches		
A	231	275	506	0.37	20.2	20	34	8,600	0.38	+ 5,150 - 4,600	+ 3,600 - 5,100	0.0516	0.0495	0.41
C	535	845	1380	1.00	55.2	19	28	22,800	1.00	+ 12,900 - 8,400	+15,600 - 5,000	.1220	.1218	1.00
E	250	364	614	0.44	24.6	22	50	9,900	0.43	+ 5,600 - 3,640	+ 5,000 - 3,000	.0498	.0467	0.38
Total	1016	1484	2500		100.0									

* Stringer

Properties of Sections Used in Computations:

Plate in T-Beam Action -- 4-1/4 inches

$I = 3.35 \text{ in}^4$

$S = 1.77 \text{ in}^3$ and 2.72 in^3

Exterior Stringer A

Plate in T-Beam Action -- 1-7/8 inches

$I = 2.84 \text{ in}^4$

$S = 1.67 \text{ in}^3$ and 2.00 in^3

TABLE VI

SERIES 20 - FIRST MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1	Fixity Factor Right K_2	Computed Center Moment	Ratio to Next Str.*	Center Stress Computed	Center Stress Measured	Center Defl. Computed	Center Defl. From Slope Reads.	Ratio to Next Str.*
						per cent		in-lb		lb per sq in		inches		
A	595	632	1228	1.00	49.1	45	50	18,900	1.00	+11,650 -11,680	+12,600 - 7,400	0.1120	0.1115	1.00
C	512	528	1040	0.85	41.6	26	25	18,100	0.96	+10,640 - 8,790	+12,000 - 5,300	.1018	.1012	0.91
E	101	133	234	0.22	9.3	29	45	3,810	0.21	+ 2,240 - 1,850	+ 860 - 2,400	.0210	.0202	0.20
Total	1208	1293	2501		100.0									

* Stringer

Properties of Sections Used in Computations:
Plate in T-Beam Action -- 2 inches
$I = 2.93\ in^4$
$S = 1.70\ in^3$ and $2.06\ in^3$

Exterior Stringer A
Plate in T-Beam Action -- 1/2 inch
$I = 2.53\ in^4$
$S = 1.62\ in^3$ and $1.618\ in^3$

TABLE VII

SERIES 21 - FIRST MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1 (per cent)	Fixity Factor Right K_2 (per cent)	Computed Center Moment (in-lb)	Ratio to Next Str.*	Center Stress Computed (lb per sq in)	Center Stress Measured (lb per sq in)	Center Defl. Com-puted (inches)	Center Defl. From Slope Reads. (inches)	Ratio to Next Str.*
A	717	918	1635	1.00	65.4	32	55	25,100	1.00	+15,500 -15,500	+20,000 - 9,200	0.161	0.149	1.00
C	336	350	686	0.42	27.4	33	36	11,130	0.44	+ 6,550 - 5,400	+ 5,700 - 5,400	0.062	0.061	0.41
E	89	86	175	0.25	7.0	51	57	2,500	0.23	+ 1,470 - 1,210	0 - 2,300	0.013	0.011	0.18
Total	1142	1354	2496		99.8									

* Stringer

Properties of Sections Used in Computations:

Plate in T-Beam Action -- 2 inches

$I = 2.93 \text{ in}^4$

$S = 1.70 \text{ in}^3$ and 2.06 in^3

Exterior Stringer A

Plate in T-Beam Action -- 1/2 inch

$I = 2.53 \text{ in}^4$

$S = 1.62 \text{ in}^3$ and 1.62 in^3

TABLE VIII

SERIES 22 - FIRST MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1	Fixity Factor Right K_2	Computed Maximum Moment	Ratio to Next Str.*	Stress at Load Computed	Stress at Load Measured	Deflection Under Load Computed	Deflection Under Load From Slope Reads.	Ratio to Next Str.*
						per cent		in-lb		lb per sq in		inches		
C	23	75	98	0.08	3.9		17	1490	0.11	+ 830 - 485	+ 1,200 - 1,500	0.006	0.006	0.12
E	341	853	1194	1.00	47.8	62	33	13,840	1.00	+ 7,740 - 4,520	+ 8,100 - 2,700	.051	.050	1.00
G	277	853	1130	0.95	45.2	47	15	14,150	1.02	+ 7,900 - 4,620	+ 8,100 - 2,600	.047	.051	1.00
I	25	53	78	0.07	3.1			1,520	0.11	+ 850 - 495	+ 600 - 2,000	.005	.009	0.18
Total	666	1834	2500		100.0									

* Stringer

Properties of Section Used in Computations:

Plate in T-Beam Action -- 5-1/2 inches

$I = 3.55 \text{ in}^4$

$S = 1.79 \text{ in}^3$ and 3.07 in^3

TABLE IX

SERIES 24 - FIRST MODEL

Floor Beam	Shear		Load	Per Cent Total Load	Fixity Factor		Computed Maximum Moment	Center Stress		Center Deflection		
	Left	Right			Left K_1	Right K_2		Computed	Measured	Meas-ured	Com-puted	From Slope Reads.
					per cent		in-lb	lb per sq in		inches		
M	1250	1250	2500	100.0	0	0	31,800	+ 9,080 - 3,450	+ 8,000 - 2,900	0.0214	0.0236	0.0216

Properties of Section Used in Computations:

Plate in T-Beam Action -- 16 inches

$I = 10.4 \text{ in}^4$

$S = 3.51 \text{ in}^3$ and 9.12 in^3

TABLE X

SERIES 32 - SECOND MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1 (per cent)	Fixity Factor Right K_2 (per cent)	Computed Center Moment (in-lb)	Ratio to Next Str.*	Center Stress Computed (lb per sq in)	Center Stress Measured (lb per sq in)	Center Defl. Computed (inches)	Center Defl. From Slope Reads. (inches)	Ratio to Next Str.*
C	44	52	96	0.37	3.8	6	31	1,742	0.39	+ 990 - 660	+ 500 - 1,200	0.0092	0.0089	0.41
D	129	134	263	0.50	10.5	23	35	4,480	0.45	+ 2,590 - 1,730	+ 1,350 - 2,200	.0226	.0218	0.44
E	263	258	521	0.68	20.8	7	23	10,075	0.77	+ 5,720 - 3,820	+ 6,300 - 2,450	.0508	.0500	0.76
F	352	416	768	1.00	30.7	19	37	13,125	1.00	+ 7,470 - 5,000	+ 9,800 - 3,000	.0672	.0659	1.00
G	233	238	471	0.61	18.8	10	25	8,590	0.65	+ 4,870 - 3,250	+ 6,400 - 2,500	.0440	.0440	0.67
H	109	124	233	0.50	9.4	18	43	3,930	0.46	+ 2,230 - 1,490	+ 2,500 - 1,500	.0201	.0190	0.43
I	49	49	98	0.42	3.9	0	40	1,785	0.45	+ 1,015 - 677	+ 300 - 1,000	.0095	.0090	0.47
Total	1179	1271	2450		97.9									

* Stringer

Properties of Sections Used in Computations:

Plate in T-Beam Action -- 4 inches

$I = 3.30 \text{ in}^4$

$S = 1.76 \text{ in}^3$ and 2.64 in^3

TABLE XI

SERIES 33 - SECOND MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1 (per cent)	Fixity Factor Right K_2 (per cent)	Computed Center Moment (in-lb)	Ratio to Next Str.*	Center Stress Computed (lb per sq in)	Center Stress Measured (lb per sq in)	Center Defl. Computed (inches)	Center Defl. From Slope Reads. (inches)	Ratio to Next Str.*
C	78	85	163	0.38	6.4	8	45	2,785	0.40	+ 1,580 - 1,050	+ 800 - 1,580	0.0149	0.0141	0.43
D	223	203	426	0.60	16.7	36	40	6,885	0.56	+ 3,910 - 2,610	+ 4,000 - 3,000	.0331	.0327	0.54
E	336	376	712	1.00	27.9	22	34	12,250	1.00	+ 7,170 - 4,640	+ 8,200 - 2,800	.0615	.0607	1.00
F	302	371	673	0.95	26.3	19	33	11,675	0.95	+ 6,640 - 4,420	+11,000 - 3,250	.0595	.0588	0.97
G	188	198	386	0.57	15.1	21	40	6,535	0.56	+ 3,710 - 2,470	+ 3,400 - 2,800	.0330	.0322	0.55
H	90	104	194	0.50	7.6	34	60	2,945	0.45	+ 1,670 - 1,115	+ 800 - 1,500	.0142	.0135	0.42
Total	1217	1337	2554		100.0									

Properties of Section Used in Computations:

Plate in T-Beam Action -- 4 inches

$I = 3.30 \text{ in}^4$

$S = 1.76 \text{ in}^3$ and 2.64 in^3

TABLE XII

SERIES 34 - SECOND MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1 (per cent)	Fixity Factor Right K_2 (per cent)	Computed Center Moment (in-lb)	Ratio to Next Str.*	Center Stress Computed (lb per sq in)	Center Stress Measured (lb per sq in)	Center Defl. Computed (inches)	Center Defl. From Slope Reads. (inches)	Ratio to Next Str.*
D	129	143	272	0.59	10.8	27	55	4,325	0.48	+ 2,460 - 1,640	+ 2,100 - 1,550	0.0213	0.0201	0.42
E	257	203	460	0.48	18.3	4	5	9,025	0.59	+ 5,125 - 3,420	+ 6,300 - 3,000	.0478	.0480	0.66
F	480	485	965	1.00	38.4	43	41	15,260	1.00	+ 8,670 - 5,780	+11,400 - 2,800	.0708	.0729	1.00
G	232	307	539	0.56	21.4	16	33	9,430	0.62	+ 5,350 - 3,570	+ 6,300 - 3,400	.0500	.0477	0.65
H	129	148	277	0.51	11.0	16	56	4,010	0.42	+ 2,280 - 1,520	+ 2,200 - 1,800	.0233	.0214	0.45
Total	1227	1286	2513		99.9									

* Stringer

Properties of Sections Used in Computations:

Plate in T-Beam Action -- 4 inches

$I = 3.30 \text{ in}^4$

$S = 1.76 \text{ in}^3$ and 2.64 in^3

TABLE XIII

SERIES 35 - SECOND MODEL

Str.*	Shear Left	Shear Right	Load	Ratio to Next Str.*	Per Cent Total Load	Fixity Factor Left K_1	Fixity Factor Right K_2	Computed Center Moment	Ratio to Next Str.*	Center Stress Computed	Center Stress Measured	Center Defl. Computed	Center Defl. From Slope Reads.	Ratio to Next Str.*
						per cent		in-lb		lb per sq in		inches		
A	650	684	1334	1.00	53.0	40	43	20,900	1.00	+12,500 -11,000	+16,200 - 6,400	0.1190	0.1206	1.00
B	326	407	733	0.55	29.0	11	41	12,620	0.60	+ 7,270 - 6,550	+ 7,100 - 5,500	.0700	.0651	0.55
C	128	156	284	0.40	11.3	0	60	5,110	0.41	+ 2,940 - 2,640	+ 1,400 - 3,700	.0274	.0270	0.41
D	71	100	171	0.60	6.9	21	65	2,660	0.52	+ 1,530 - 1,380	+ 600 - 2,400	.0143	.0129	0.48
Total	1175	1347	2522		100.2									

* Stringer

Properties of Sections Used in Computations:

Plate in T-Beam Action -- 3 inches

$I = 3.13 \text{ in}^4$

$S = 1.74 \text{ in}^3$ and 1.93 in^3

Exterior Stringer A

Plate in T-Beam Action -- 1-1/2 inches

$I = 2.78 \text{ in}^4$

$S = 1.67 \text{ in}^3$ and 1.90 in^3

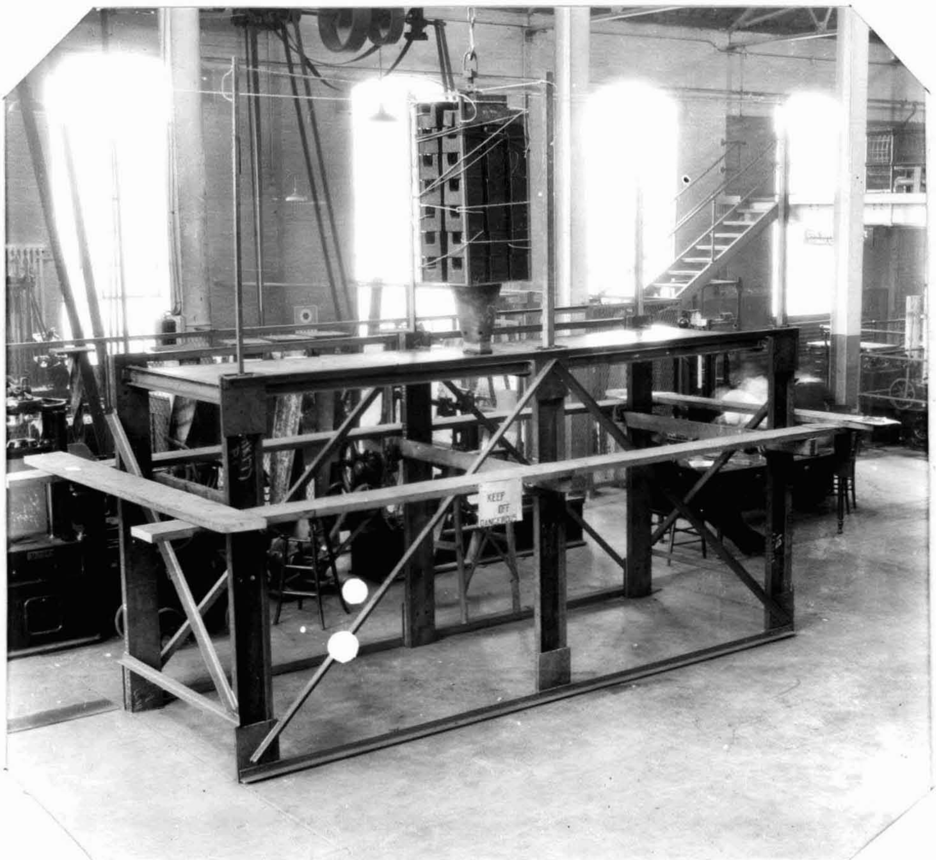


Fig. 1 - Photograph of First Model Floor

and Loading Rig

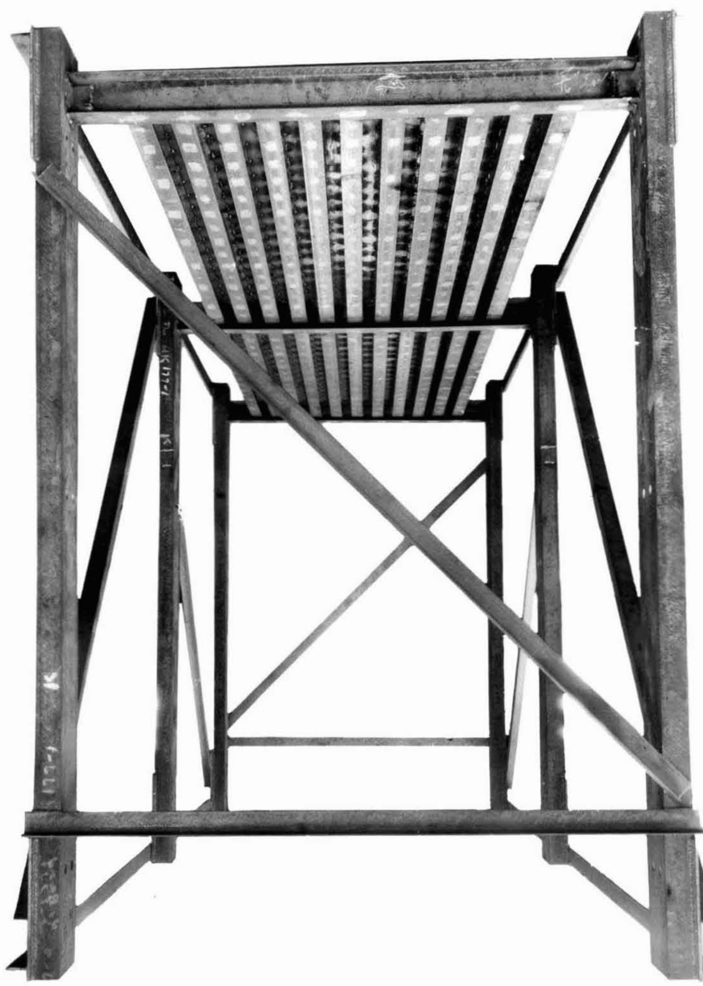


Fig. 2 - Bottom View Of Floor

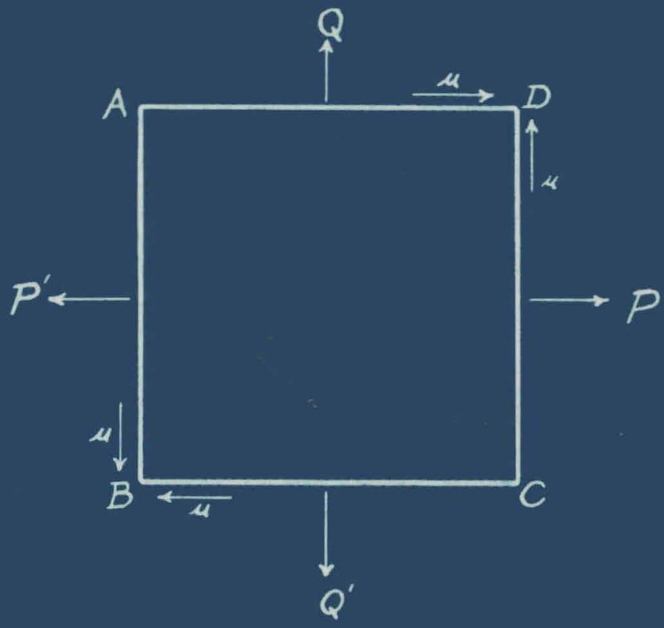


Fig. 3

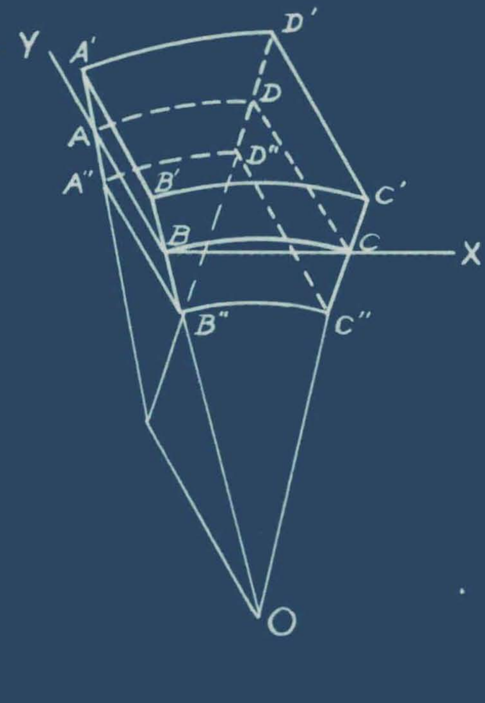


Fig. 4

Tensile Test of Plate

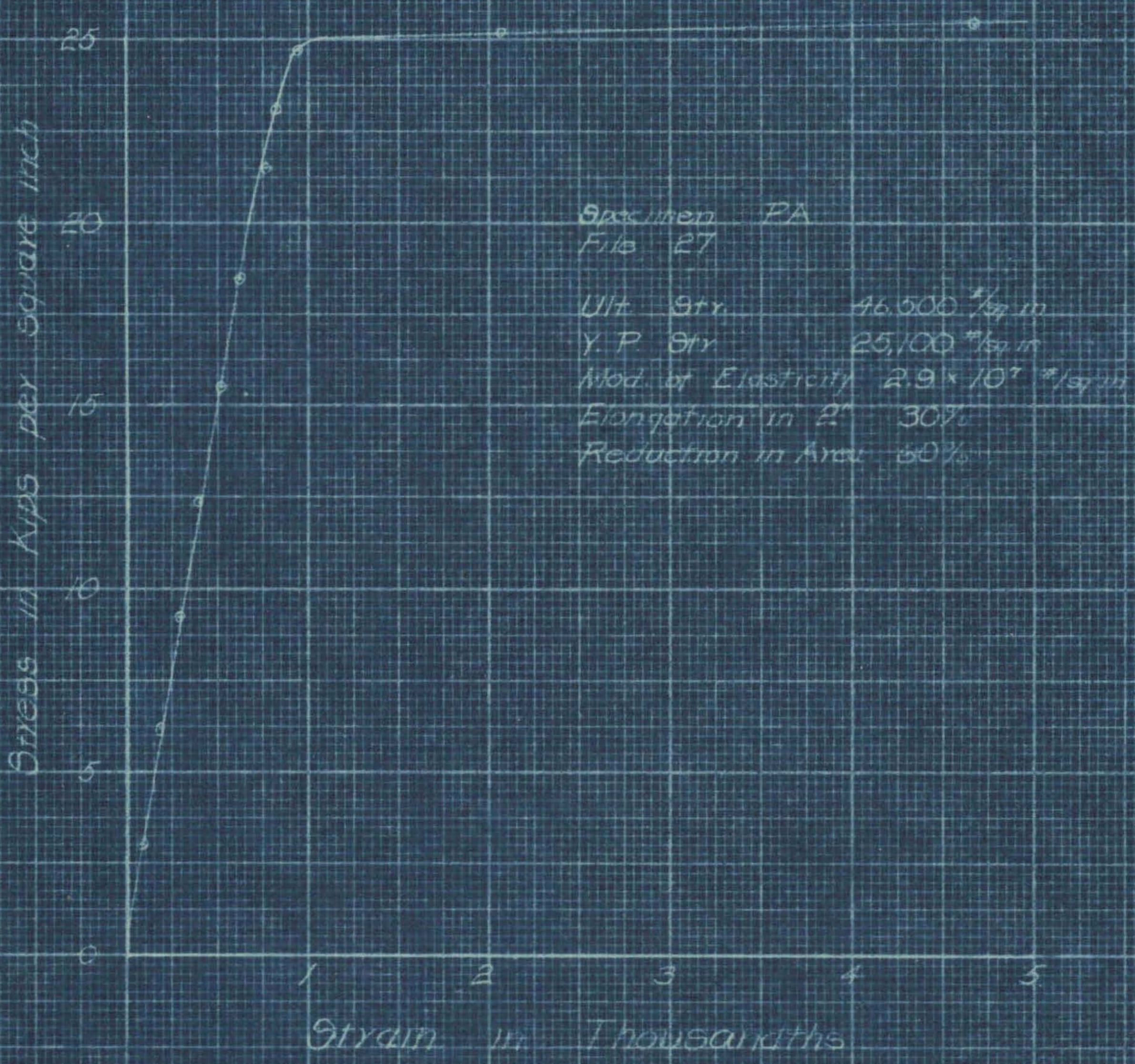


Fig. 5

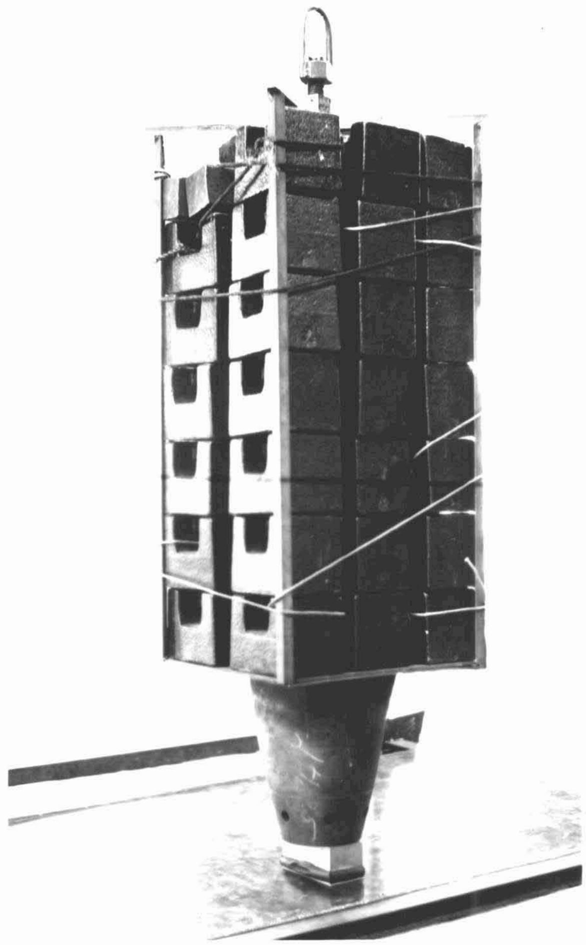
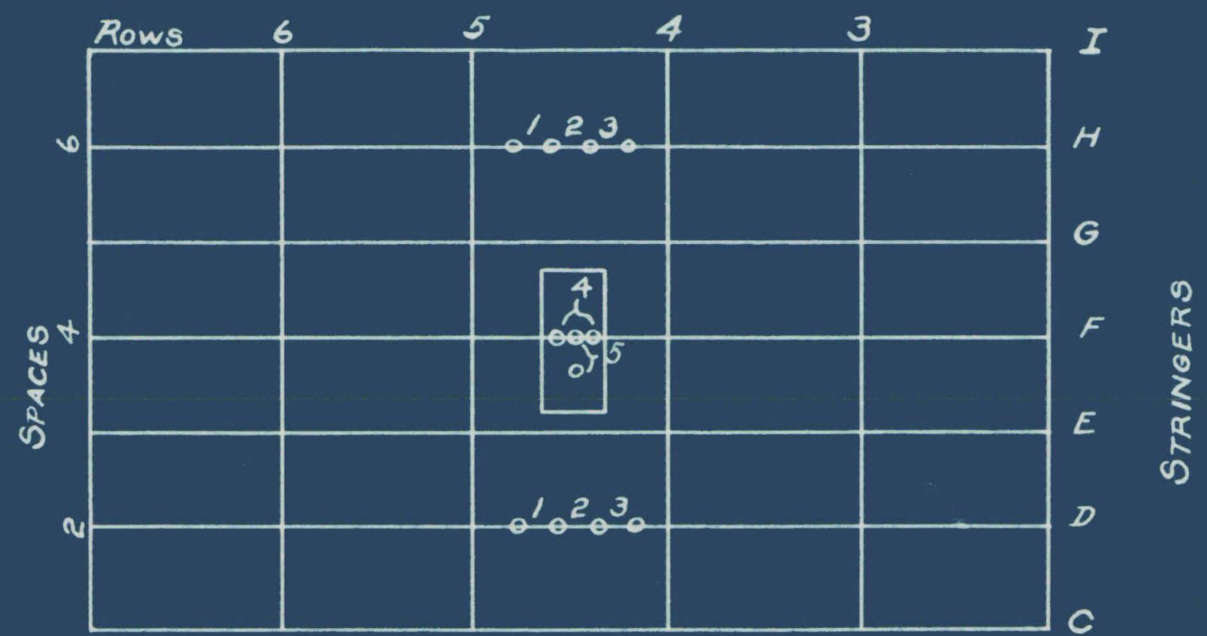


Fig. 6 - Loading Rig



Sketch Showing Gage Holes

For Load Comparison

Fig. 7

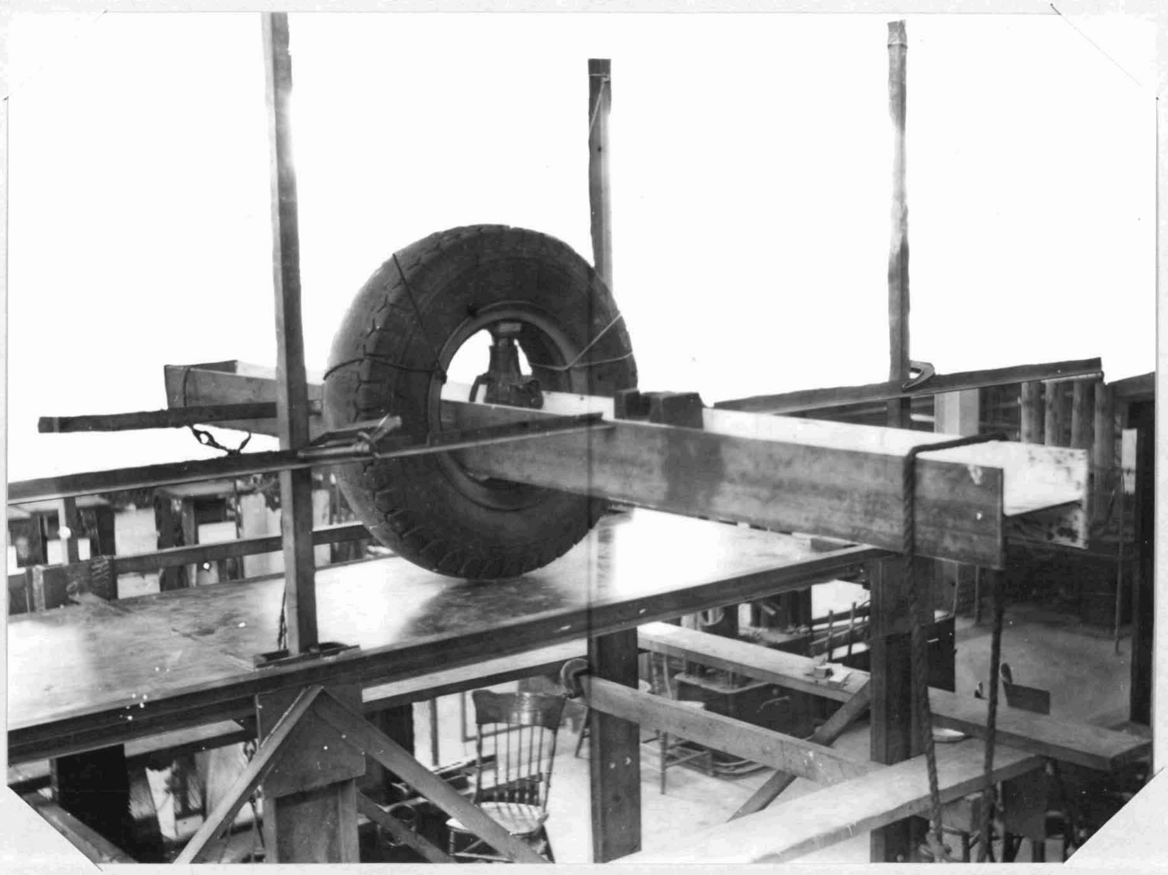


Fig. 8 - Loading Floor Through Tire

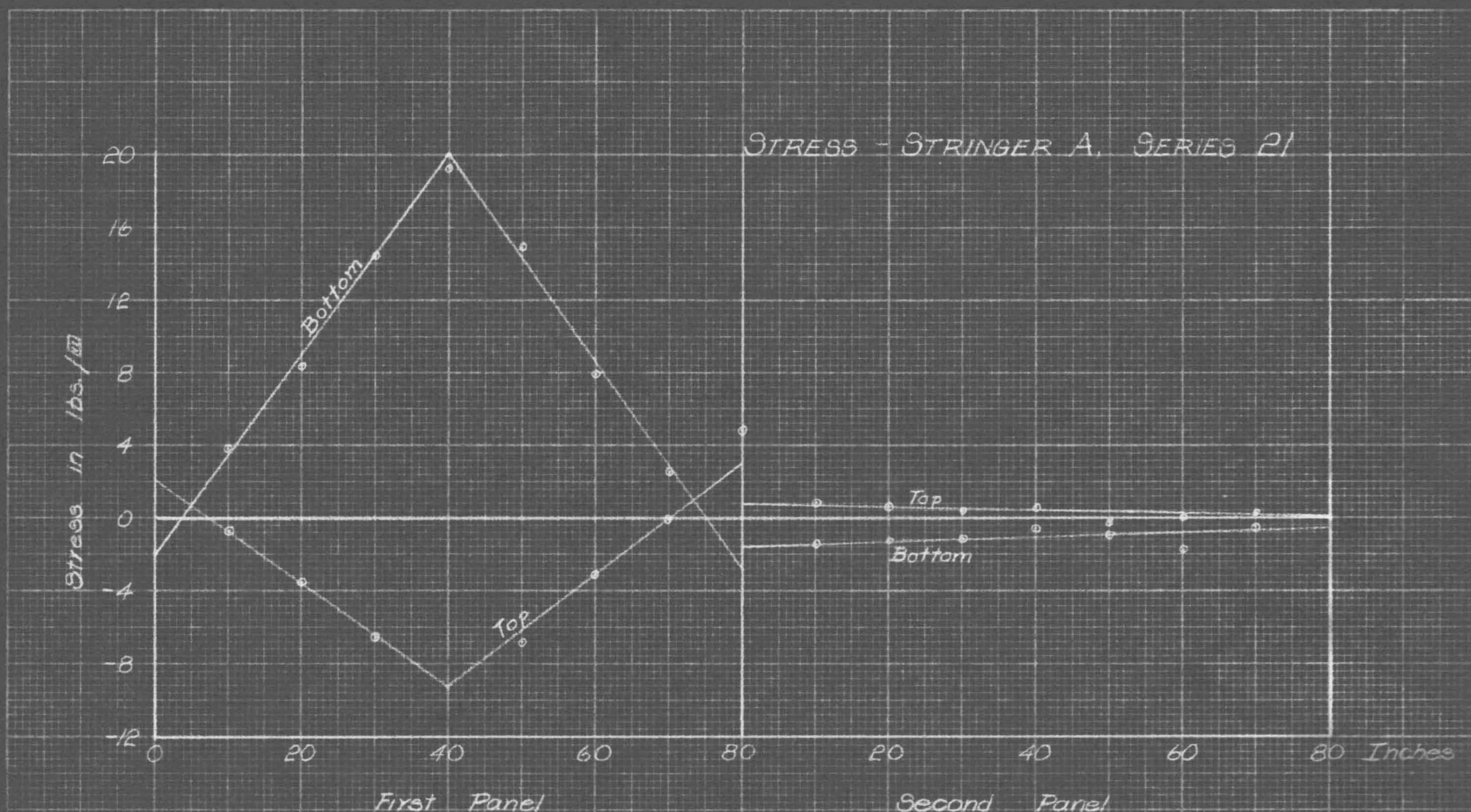

STRESS - STRINGER A, SERIES 21
Stress in lbs./in
20
16
12
8
4
0
-4
-8
-12
Bottom
Top
0
20
40
60
80
Inches
First Panel
Second Panel

Fig. 9

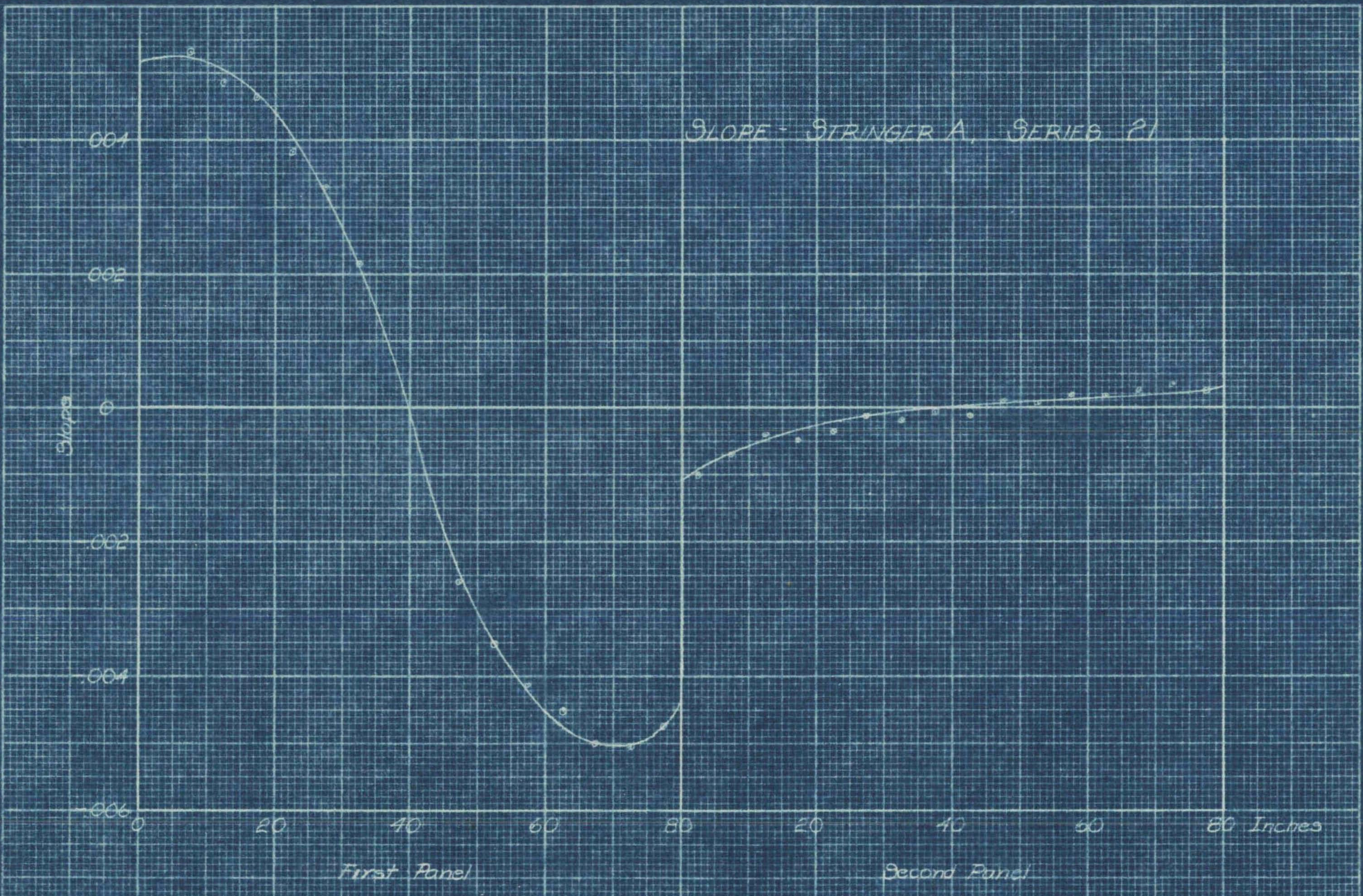

SLOPE - STRINGER A, SERIES 21
.004
.002
0
.002
.004
.006
Slope
0
20
40
60
80
20
40
60
80 Inches
First Panel
Second Panel

Fig. 10

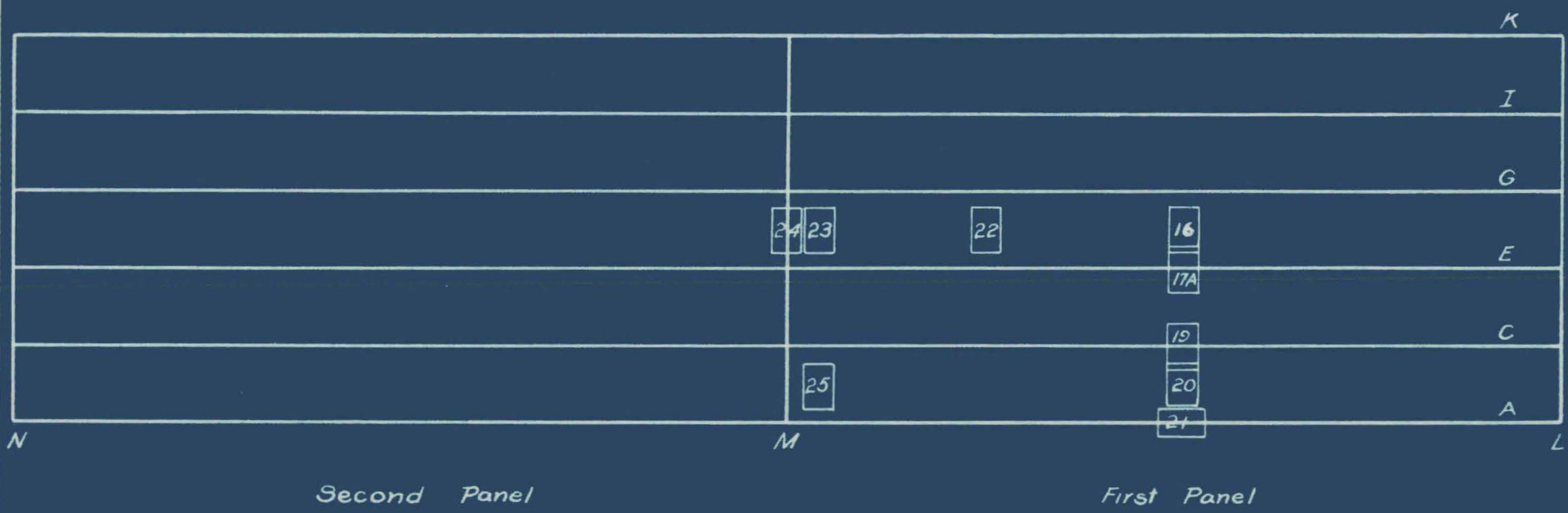


Fig. 11

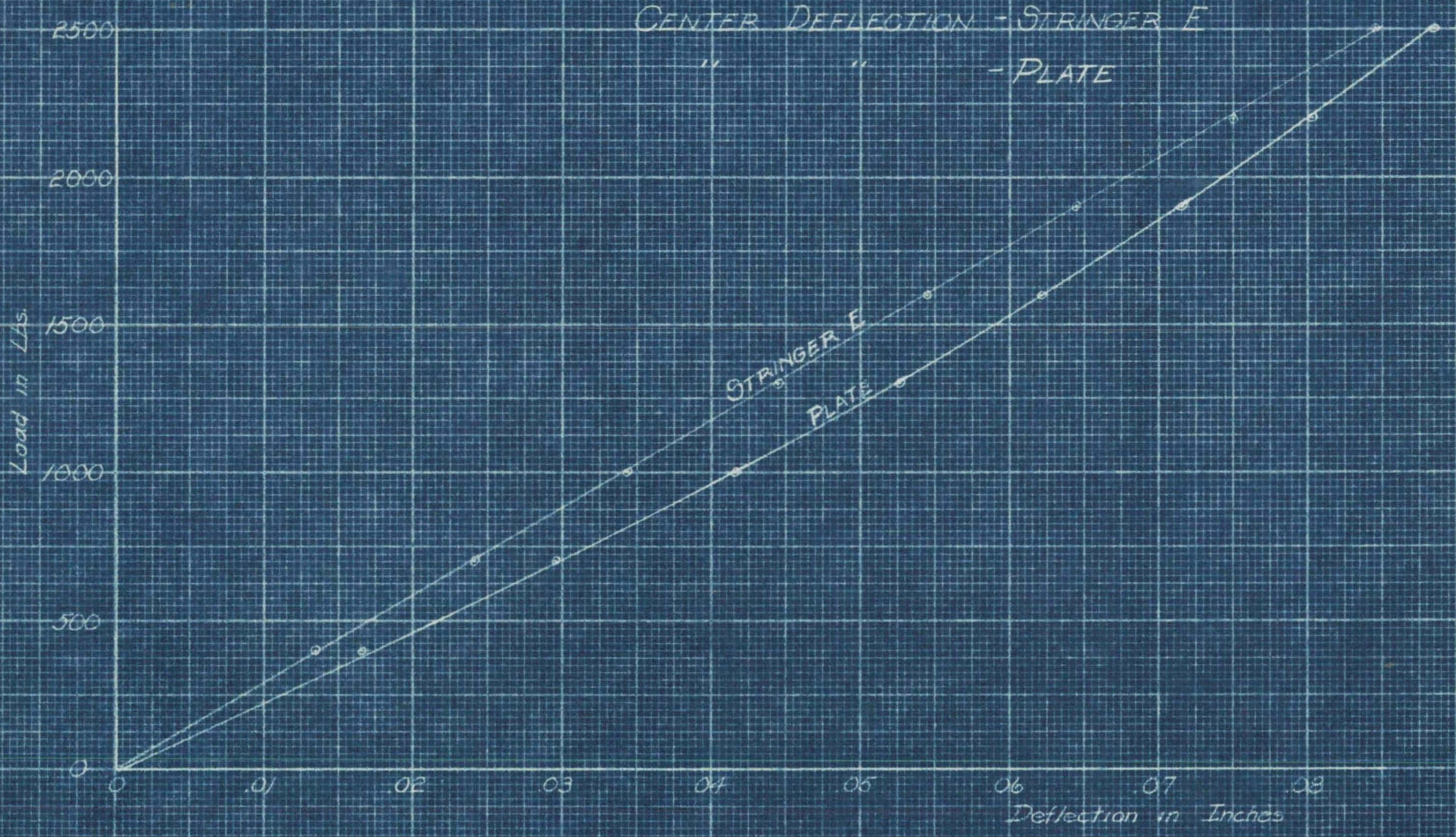

CENTER DEFLECTION - STRINGER E
" " - PLATE
STRINGER E
PLATE
2500
2000
1500
1000
500
0
Load in Lbs.
0
.01
.02
.03
.04
.05
.06
.07
.08
Deflection in Inches

Fig. 12

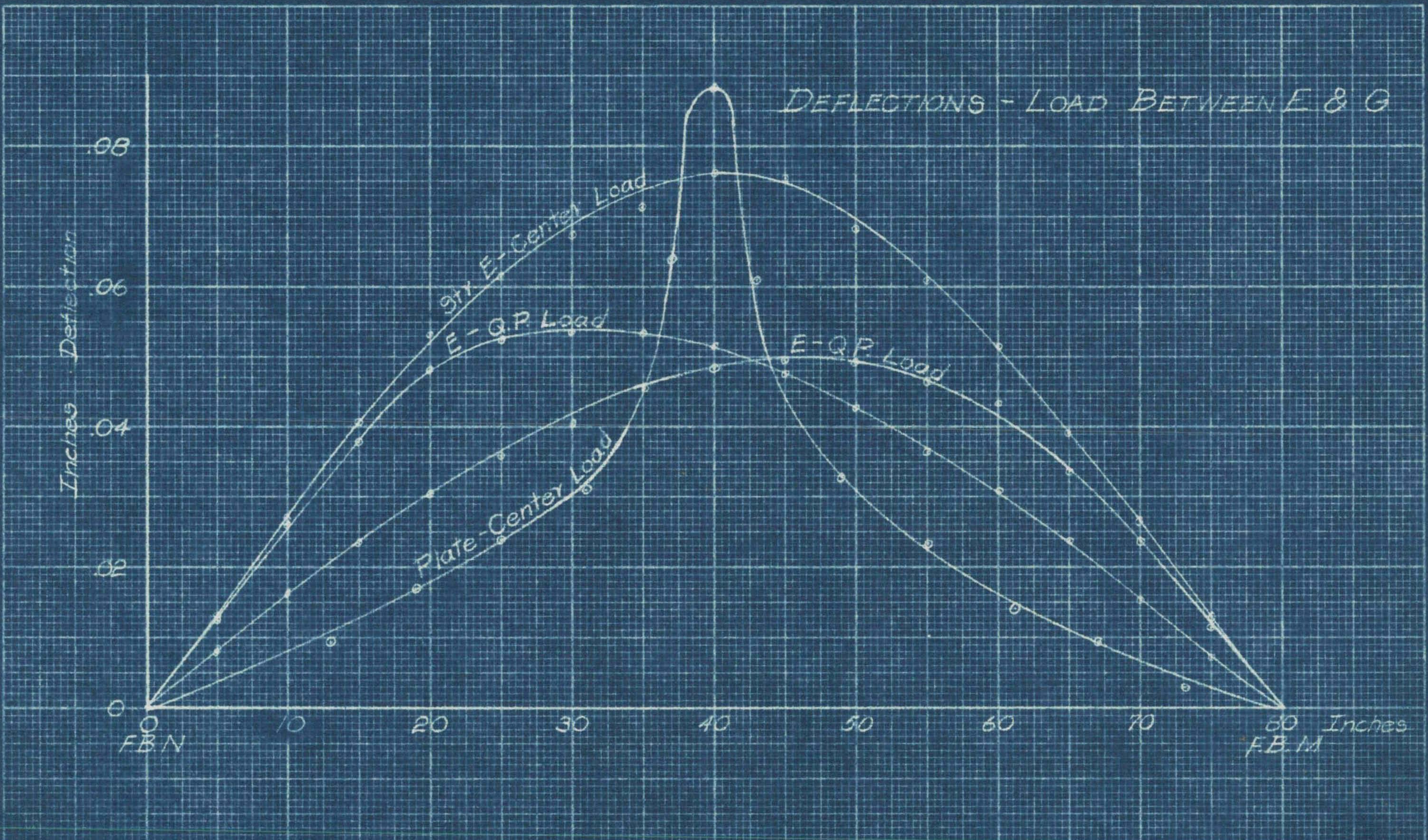

DEFLECTIONS - LOAD BETWEEN E & G
Inches Deflection
.08
.06
.04
.02
0
0
10
20
30
40
50
60
70
80
Inches
F.B.N
F.B.M
Str. E-Center Load
E-Q.P. Load
E-Q.P. Load
Plate-Center Load

Fig. 13

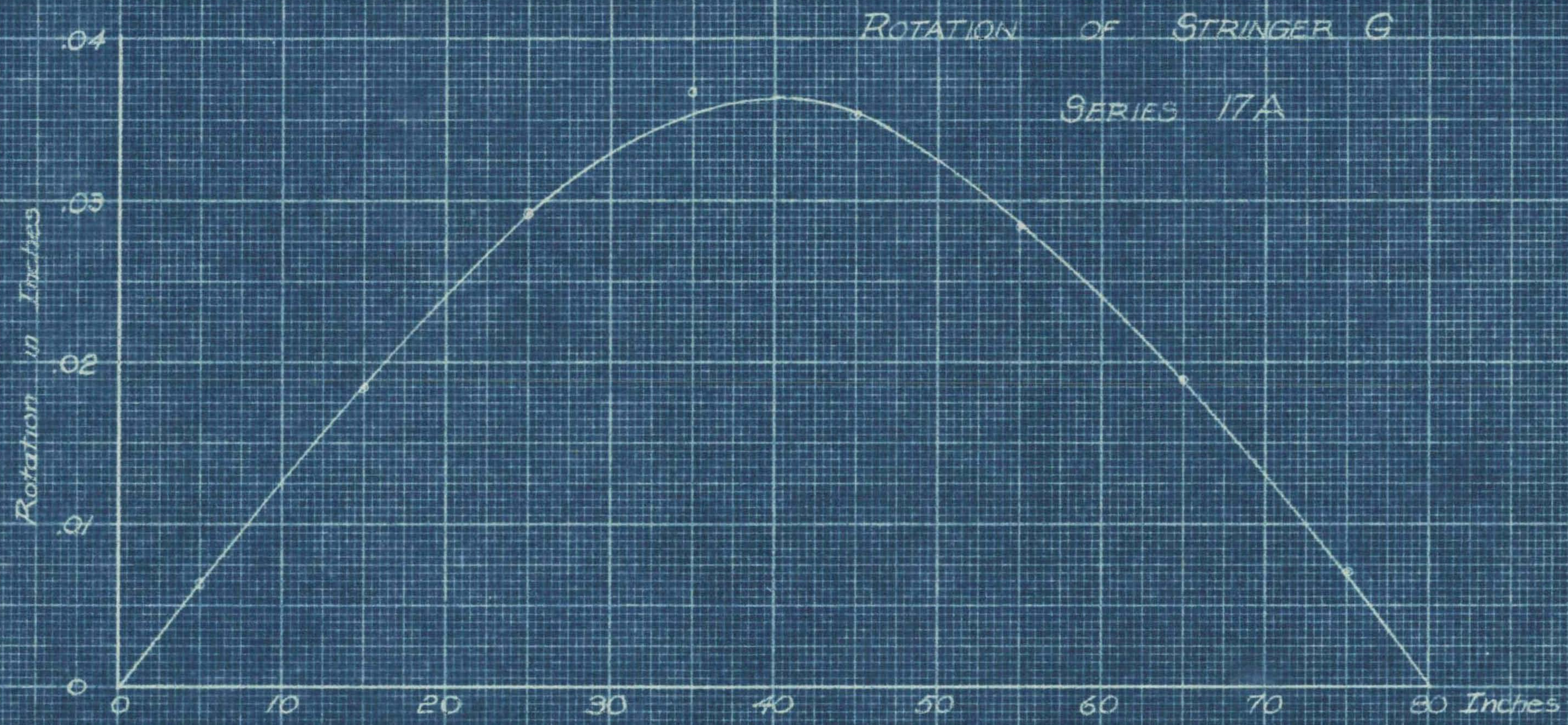

ROTATION OF STRINGER G
SERIES 17A
Rotation in Inches
.04
.03
.02
.01
0
0
10
20
30
40
50
60
70
80 Inches

Fig. 14

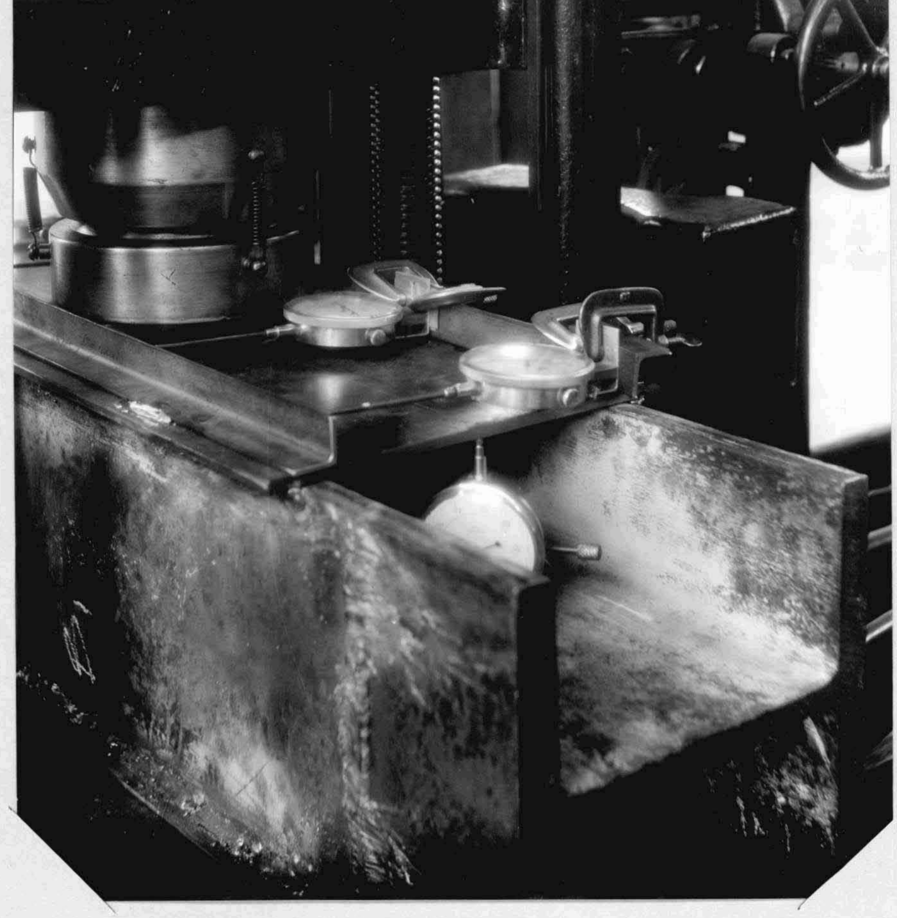


Fig. 15 - Plate Ready For Test In Machine

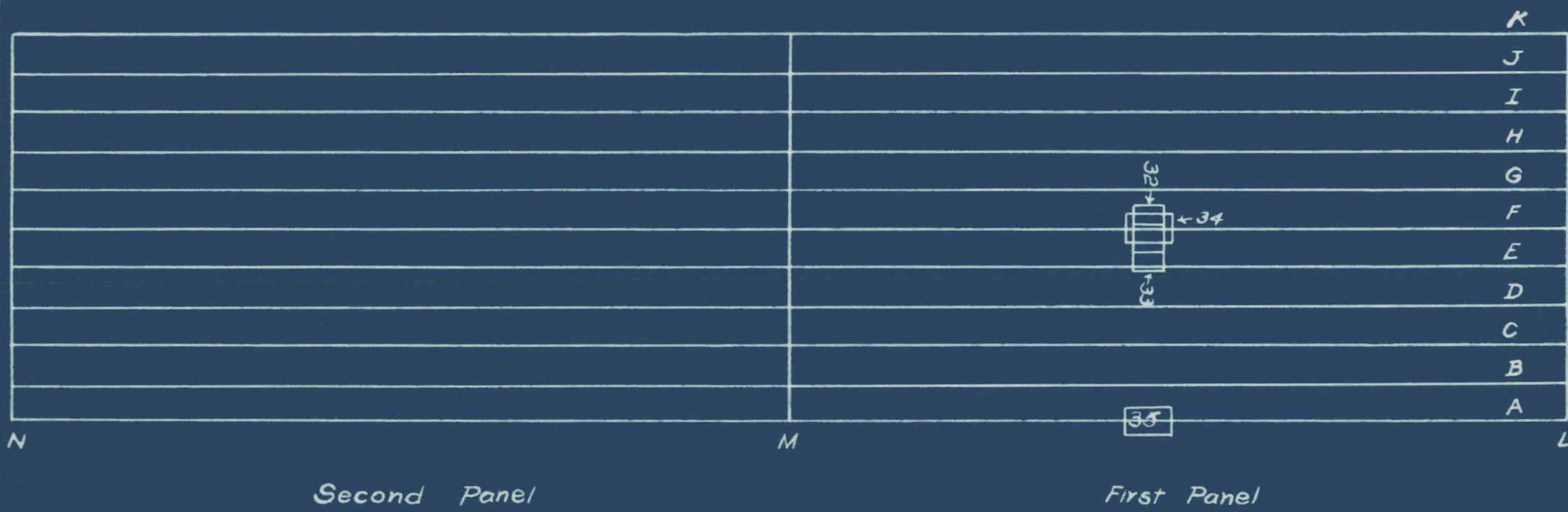


Fig. 16

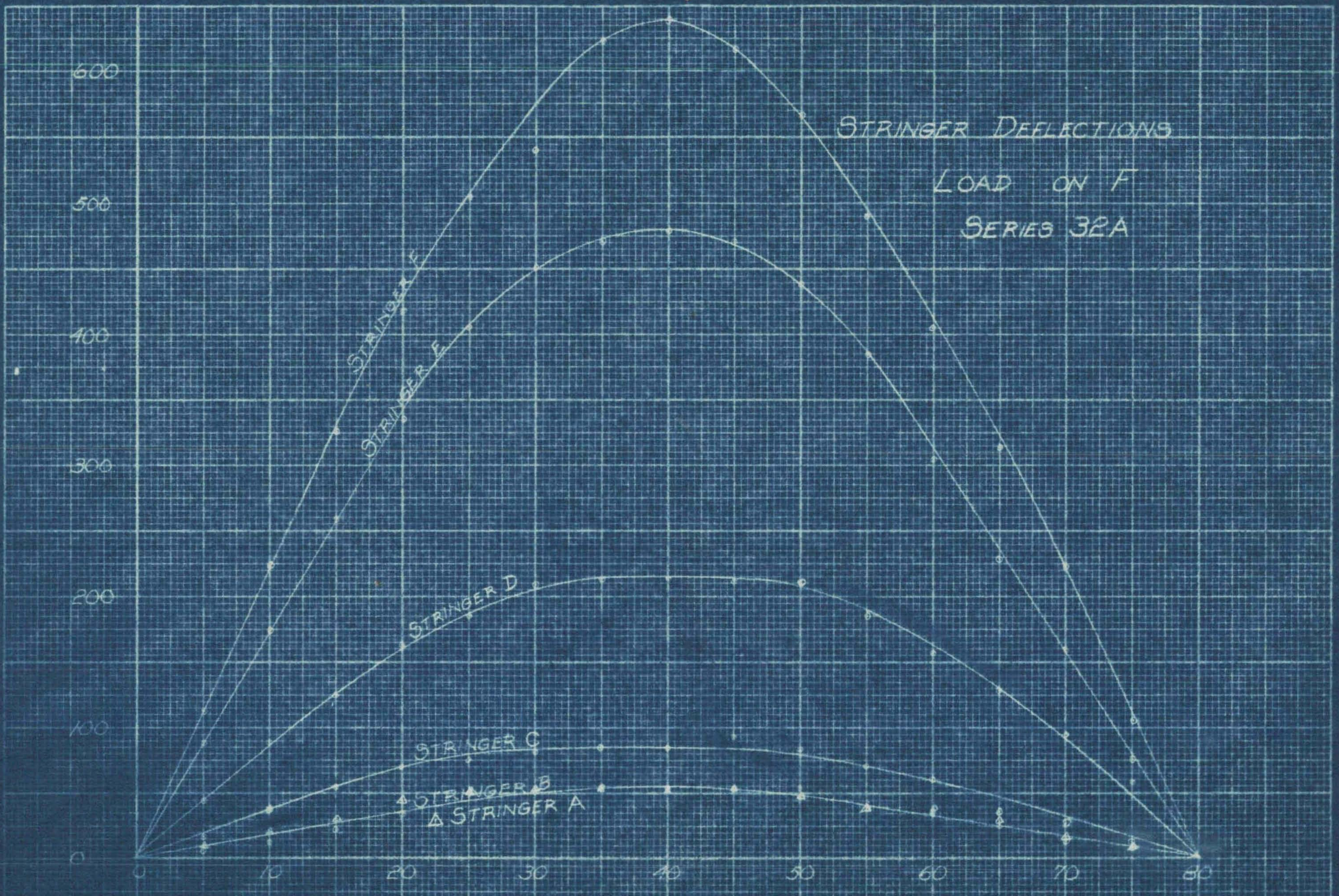
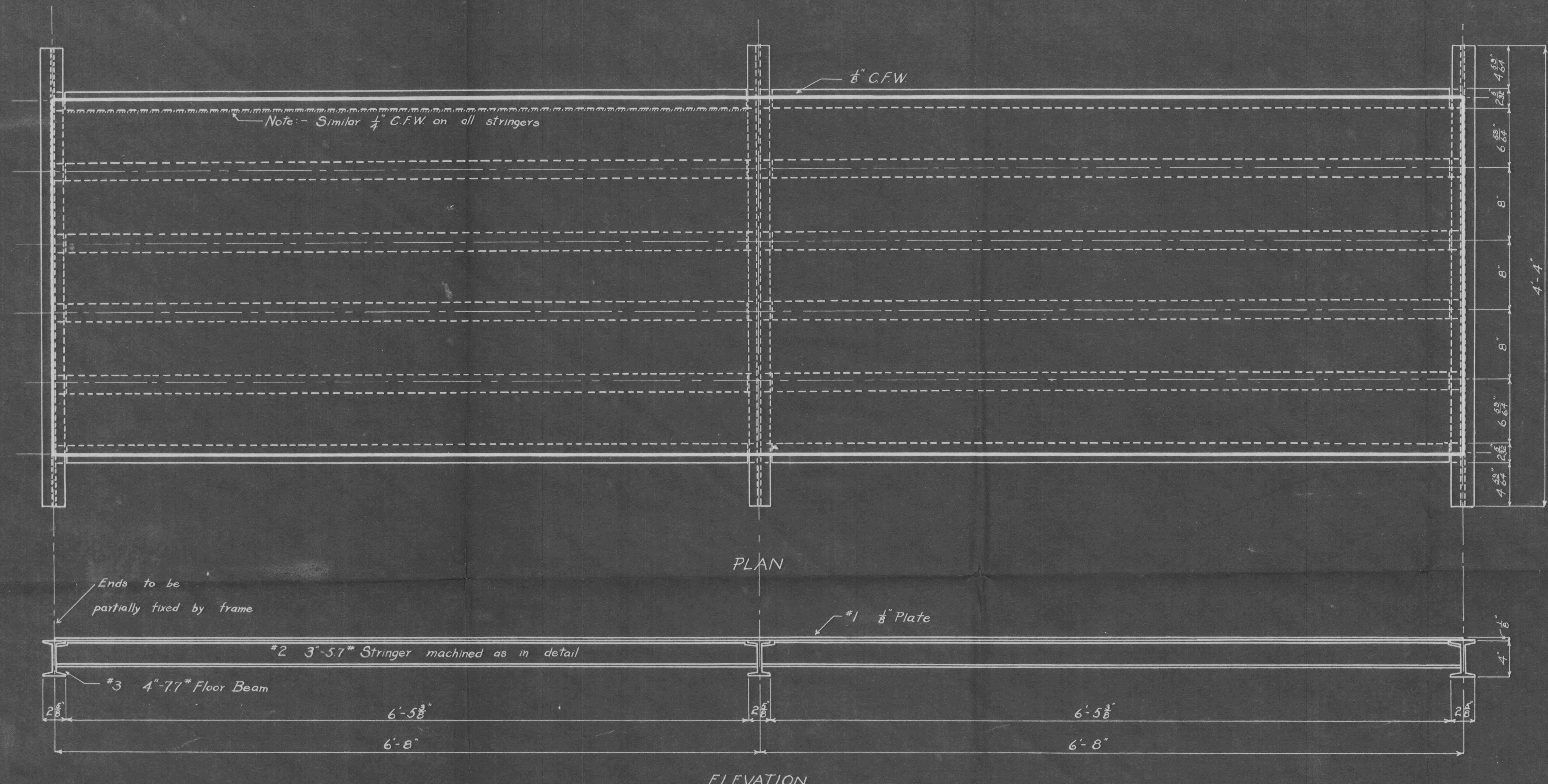
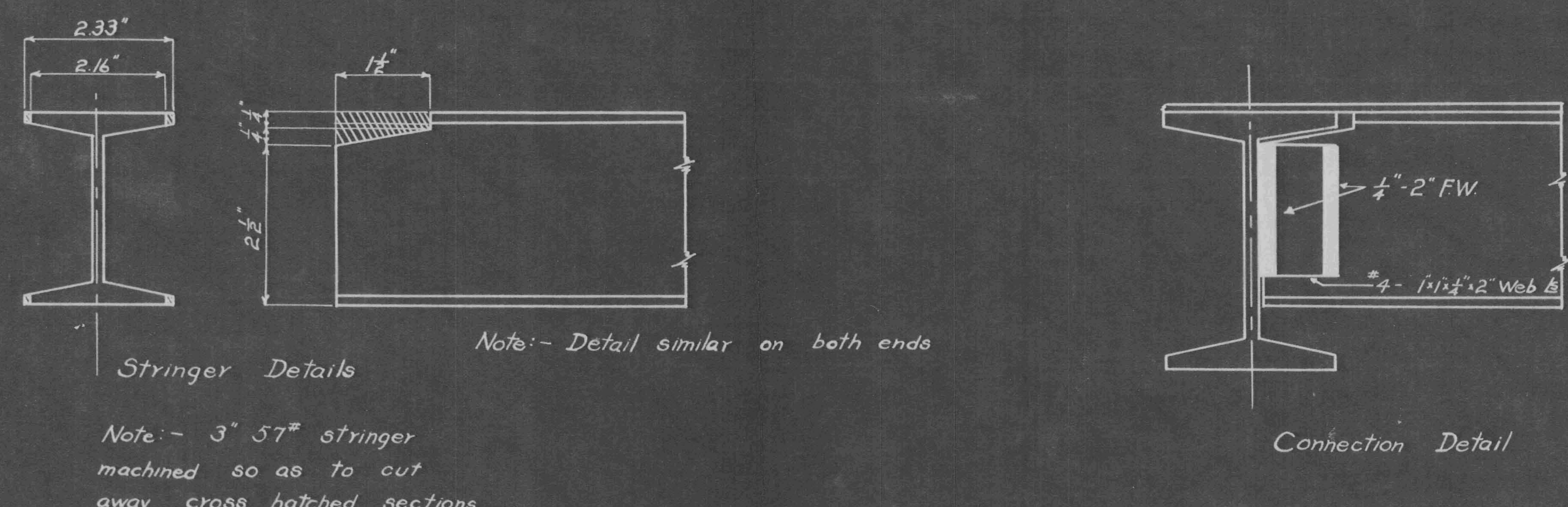

STRINGER DEFLECTIONS
LOAD ON F
SERIES 32A
STRINGER F
STRINGER E
STRINGER D
STRINGER C
STRINGER B
STRINGER A
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80

Fig. 17





No.	Quan	Size	Article
1	1	13'-4" x 3'-4" x $\frac{1}{8}$"	Floor Plate (Flame Cut)
2	22	6'-7$\frac{1}{2}$" $\pm\frac{1}{32}$"	3"-5.7# Stringers as in detail
3	3	4'-4"	4"-7.7# Floor Beams
4	88	1"x1"x$\frac{1}{4}$"x2"	Angles

MODEL #1

BATTLEDECK FLOOR TESTS

Scales:
1" = 18"
1" = 2"

Drawn by
I. Madsen